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Multi-Task Simultaneous Supervision: Dual Resource-Constrained Scheduling Problem in Identical Parallel Machines Considering Differences in Operator Skill Levels

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Abstract: This paper focuses on developing a Multi-Task Simultaneous Supervision Dual Resource-Constrained Scheduling (MTSSDRC) system that considers differences in skill between operators, aiming to minimize *makespan* and balance operator workload. Workload balance is calculated using the Workload Smoothness Index (WSI). The mathematical model developed uses three techniques: Mixed-Integer Linear Programming (MILP), Mixed-Integer Quadratic Programming (MIQP), and Mixed-Integer Quadratically Constrained Programming (MIQCP). These techniques can handle scheduling cases on a small to medium scale. Results from MILP focus on minimizing *makespan*, with an additional constraint for calculating the WSI. MIQP focuses on workload balance so that the WSI value becomes an objective function. It also adds a constraint for the allowable *makespan* value. The result from MIQP shows that the WSI value is lower than in MILP, and the *makespan* values are equal to the MILP *makespan* value. Next, MIQCP aims to minimize *makespan* with a constraint for the allowable WSI value. The MIQCP model produces a *makespan* value adjusted to a WSI value close to zero. Finally, further analysis is presented regarding the influence of differences in operator skills based on the results of the three models. Based on these models, operators with better skills will be assigned more frequently than others.

Keywords: MTSSDRC scheduling, identical parallel machines, workload smoothness index, *makespan*.

Introduction

Aircraft manufacturing is a complex industrial sector that requires high precision in each aspect of its operations. Production scheduling is an essential component that determines efficiency in this industry. This industry involves multiple production processes that must be coordinated within an appropriate timeframe. To increase worker productivity, some companies assign each operator to supervise two or three semi-automatic machines simultaneously. This practice aims to increase operator activity and reduce unproductive time. Moreover, many factors contribute to the increasing complexity of scheduling, including machine and human factors and differences in operator skills. Variations in operator skills lead to differences in job completion times between operators performing the same job [1], [2].

These conditions are particularly relevant in an aircraft manufacturer using Cincinnati Milacron Double Gantry All Purpose (CM-DGAP) and Cincinnati Milacron Double Gantry Aluminum (CM-DGAL) machines. These machines produce aluminum components on the shop floor and are semi-automatic, capable of performing several activities (such as machining) while other tasks still require operator assistance [3]. The machines are arranged in parallel and exhibit identical performance characteristics (e.g., capability, speed, and capacity). Operators on the shop floor must be capable of using all machines, though their skill levels naturally vary, leading to differences in job completion times for activities like setup and unloading.

Most scheduling problems consider only machines as constraints in schedule planning. However, operators can also be constrained in natural systems due to their availability and skill levels [4]. Therefore, machines and operators become priority constraints in planning the schedule, known as dual resource constraints (DRC) [5], [6].

In DRC scenarios, operators can use all machines on the shop floor [5], [7]. Typically, the number of available operators is less than the number of available machines [8], [9]. This constraint is addressed in research by Agnetis et al. [10], [11] and Mencía et al. [12], focusing on minimizing *makespan*, and by Berti et al. [13] and Shahvari & Logendran [4], focusing on minimizing production costs. According to Zouba et al. [14], operators must supervise machines simultaneously because there are fewer operators than machines. Hence, operators can move from one machine to another to supervise multiple jobs simultaneously. Akbar and Irohara [15] studied simultaneous supervision, where operators can simultaneously operate multiple machines with two activity elements (setup and unloading). This scheduling research is the Multi-Task Simultaneous Supervision Dual Resource-Constrained (MTSSDRC) Scheduling Problem. Assigning operators to multiple machines simultaneously can increase machine waiting and operator idle time, affecting the *makespan* [15]–[19]. Therefore, minimizing *makespan* is one of the objectives of this research.

This research aims to produce results applicable to natural manufacturing environments considering human factors. The human factor in the MTSSDRC system is viewed from a skill perspective, a numeric value assessing each operator's skill level to meet quality standards or specific productivity targets [1]. Biskup [20] was the first researcher to study the effect of skill levels on scheduling problems. Operators with varying knowledge absorption processes may require retraining or relearning [21].

Research by Kuo and Yang [22] considers differences in operator skill levels as constraints to minimize flow time in a machine. Costa et al. [1] and Vallada & Ruiz [2] also consider operator skill levels affecting setup time differences for a job performed by different operators. Inappropriate scheduling can create a workload imbalance among operators, leading to dissatisfaction and jealousy. Achieving operator workload balance is crucial to prevent such feelings in production schedule planning [19]. The goal is for the planned production schedule to provide each operator with a balanced workload.

In this paper, we develop a new MTSSDRC scheduling model that considers differences in operator skill levels. The references for this paper come from various researchers and shop floor conditions. We will develop the mathematical model using mixed-integer linear programming (MILP), mixed-integer quadratic programming (MIQP), and mixed-integer quadratically constrained programming (MIQCP), ensuring the model runs with Gurobi software. The goal is to create a mathematical model to minimize *makespan* and balance operator workload.

Methods

Literature Review

Grahan et al. [23] have classified scheduling in the manufacturing environment for decades. This classification remains relevant for identifying the nature of scheduling problems. The scheduling classification is based on a triplet $\alpha|\beta|\gamma$: machine environment (α field), processing characteristics and constraints (β field), and objective function (γ field). The $\alpha\ field$ includes *Single Machine* (I), *Identical Machines in Parallel* (P_m), *Machines in Parallel with Different Speeds* (Q_m), *Unrelated Machines in Parallel* (R_m), *Flow Shop* (F_m), *Job Shop* (J_m), and *Flexible Job Shop* (FJ_e) (Pinedo, 2012).

Scheduling with parallel machines involves planning *n* tasks that work independently on m parallel machines, considering specific constraints to optimize performance [24]–[27]. According to Biskup et al. [28], the parallel system allocates tasks to simultaneously operating machines. Each task requires one process on one of the *m* available machines [29. When all machines have the exact specifications and are arranged in parallel, they are known as Identical Parallel Machines [30]. This environment includes multiple machines with identical performances [30]. The basic model for an identical parallel machine system involves each job requiring one task or specific activity, which can be processed on any machine [31].

Processing characteristics and constraints in the β field include release dates, preemptions, sequence-dependent setup times, batch processing, permutation, and recirculation [31]. Constraints include the available resources, such as tools, machines, and operators. Often, the number of operators is not equal to the number of machines, creating a constraint when planning the production schedule. Machines and operators as resources can be potential constraints in solving scheduling problems, known as *Dual Resource Constraint Scheduling Problems* (DRCSP) [6]. DRCSP research details processing characteristics and constraints (β field), including operator job activities where operators only handle setup and unloading without performing the machining process.

Researchers have focused on setup, processing, and unloading activities. Agnetis et al. [10], [11], Li et al. [9], and Mencia [12] consider processing time (including setup and unloading time) to develop scheduling models. Costa et al. (2013), Gao & Pan (2016), Munoz et al. (2022), and Zhang et al. (2021) developed scheduling models considering *setup* time as a constraint.

DRCSP also considers differences in operator skill levels. Operators have varying skill levels, resulting in different completion times for setup and unloading activities [1], [2]. Careful production scheduling is necessary to prevent imbalances in operator assignments, which can lead to dissatisfaction and jealousy [19].

Research on DRCSP is expanding with the consideration of simultaneous supervision. Zouba et al. [14] studied m identical parallel machines with w operators to minimize *makespan*, where $w < m$. Thus, each operator must supervise multiple machines simultaneously. This research was further developed by Akbar and Irohara [15], considering an advanced optimization method: multi-task simultaneous supervision dual resource-constrained (MTSSDRC) scheduling. MTSSDRC involves machine assignment, operator allocation, and job sequencing [17]. Setup and unloading must be scheduled for the operator, who can leave the machine while machining is in progress. Assigning an operator to multiple machines alternately can increase waiting and idle time, thus increasing *makespan* [15]. Therefore, minimizing *makespan* becomes an objective function to reduce idle and waiting times in MTSSDRC scheduling [15]–[19].

The objective function (γ field) pertains to the study and theory development of scheduling for various applications [31]. It includes minimizing flow time, total tardiness, completion time, *makespan*, and production cost [31]. Minimizing *makespan* is a popular objective function in scheduling research due to its importance in assessing the effectiveness of solutions. Studies by Akbar & Irohara [17], Arroyo & Leung [35], Costa et al. [1], Peng et al. [36], Shahvari & Logendran [4] and Vallada & Ruiz [2] have used *makespan* as an objective function. In DRC scheduling, operator workload balance is also considered a secondary objective function, as seen in research by Akbar and Irohara (2018a, 2020b).

Maintaining operator workload balance is crucial to preventing jealousy over assigned workloads. Workload imbalances can occur when operator skills and scheduling are not appropriately matched [18], [19]. Based on the discussed scheduling characteristics, previous research is summarized, forming the literature basis for this study's development.

Table 1. Papers on MTSSDRC Scheduling

Literature	Problem characteristic						γ Field
	α field	DRC Type					
		w vs. m	Simultaneous Supervision	Task Type	Moving	*Operator Skill Variance*	
Hu [24]	P_m	$w > m$	-	-	-	-	*Total Tardines*
Hu [25]	P_m	$w < m$	-	-	-	-	*Total Flow Time*
Agnetis et al. [10]	J_m	$w < m$	-	-	-	-	*Makespan*
Vallada & Ruiz [2]	P_m	Any	-	-	-	√	*Makespan*
Costa et al. [1]	R_m	$w < m$	√	*Set up*	-	√	*Makespan*
Agnetis et al. [11]	J_m	$w < m$	-	-	-	-	*Makespan*
Mencía et al. [12]	J_m	$w < m$	-	-	-	-	*Makespan*
Baptiste et al.[3]	P_m	$w < m$	√	*Loading, set up, controlling, unloading*	-	-	*Makespan*
Akbar & Irohara [15]	P_m	$w < m$	√	*Set up, unloading*	√	-	*Makespan*
Akbar & Irohara [16]	P_m	$w < m$	√	*Set up, unloading*	√	-	*Makespan and operator work balance*
This research	P_m	$w < m$	√	*Set up, unloading*	√	√	*Makespan and operator work balance*

This research addresses scheduling planning problems for producers using semi-automatic machines. Detailed operator activities include setup, unloading, and moving to create schedules that minimize *makespan* and balance the operator workload. The MTSSDRC study, considering differences in operator skill levels, can be applied in natural systems.

Problem Statement

Cincinnati Milacron Double Gantry All Purpose (CM-DGAP) and Cincinnati Milacron Double Gantry Aluminum (CM-DGAL) are crucial machines for cutting and shaping aircraft components, such as aluminum rear spars, with high precision. These machines have the same capability and speed in the production process and are used by operators in fewer numbers than the machines available. Furthermore, each operator possesses different skill levels, leading to variations in activity completion times. For example, operators 1 and 2 have different times for setting up the same job.

This paper refers to research by Akbar & Irohara [17], assigning a set $I = (i_1, i_2, \dots, i_m)$ of m semi-automatic parallel machines to process a set $J = (j_1, j_2, \dots, j_n)$ of n jobs controlled by a set $K = (k_1, k_2, \dots, k_w)$ of w operators, with $w < m$ condition. Each job requires one machine to complete setup, machining, and unloading sequentially. No interruptions are allowed during the production process. Each operator is responsible for supervising setup and unloading activities. Each operator can perform any required activity for a job; for example, operator 1 sets up job 1, and operator 2 unloads job 1.

Each operator has a specific skill level, so the time to complete setup and unloading activities varies among operators [1]. The operation time for activity b of job l by operator k can be described as follows:

Where:
O_{kbl} = Operation time for activity b of job l performed by operator k
O_{bl} = The standard operation time (setup and unloading) needed by the operator during operation
η_k = Skill level coefficient of operator k

The operation time for activity b of job l performed by operator k is modeled as

$$O_{kbl} = O_{bl} \times \eta_k \tag{1}$$

O_{bl} is the standard time required for a specific activity b *job* l, and η_k is the skill level coefficient of operator k [1]. An advanced operator has a η_k value greater than 1. Skill levels are determined through direct observation in the industry environment.

The Mathematical Model

1. MILP Model

The main goal of this research is to develop a mathematical model for MTSSRC scheduling, considering differences in operator skill levels. The model uses Mixed-Integer Linear Programming (MILP) to minimize *makespan*. The developed model includes an unloading dummy activity (marked by activity 0), representing the operator's first activity to operate the machine for job 0. This is necessary to ensure the next activity is set up for the subsequent machine and job.

Input parameters for MTSSDRC involve time. The operation time is O_{kbl}. The machining time for job l is p_l. The moving time for the operator after completing a task from machine g to machine i, is t_{gi}. The objective function minimizes the time needed to optimize the completion of a set job, modeled with Eq.2.

$$minimize\ f_1 = C_{max} \tag{2}$$

We use mathematical model references from Akbar & Irohara [19], who developed the MTSSDRC model, and Costa et al. [1], who developed the DRC model considering operator skill levels and conditions on the shop floor. Thus, the mathematical model development for MTSSDRC, considering differences in skill levels, is as follows:

Indices
f, j, l = $0,1,2, \dots, n$ jobs
g, h, i = $1,2, \dots, m$ machines
k, q = $1,2, \dots, w$ workers
a, b, c = 1, 2 activities (1–setup; 2–unloading)

Parameters
P_l = Processing time for *job* l
O_{kbl} = Operation time for activity b job l by operator k

T_{gi} = Moving time between machine g and machine i
B = *Big number*

Decision variables

$X_{khajibl}$ Binary variable = 1, if the operator performs activity b of job l at machine i after completing activity a of job j at machine h.
Q_{fl} Binary variable = 1, if job l precedes job f on the same machine.
P_l^c Completion operation time for *job* l
O_{bl}^c Completion operation time for activity b of *job* l
T_{bl}^c Moving completion time before performing *activity* b of *job* l
C_{max} *Makespan*
NW_k Non-waiting time for operator k
NW_{max} Non-waiting time of the busiest operator

Mathematical models:

$$\min f_1 = C_{max} \tag{3}$$

Subject to:

$$\sum_{k=1}^{w}\sum_{h=1}^{m}\sum_{a=1}^{2}\sum_{j=0}^{n}\sum_{i=1}^{m} X_{khajibl} = 1 \;\; \forall b = 1,2; l = 1,2,\dots,n \tag{4}$$
$$\sum_{k=1}^{w}\sum_{h=1}^{m}\sum_{i=1}^{m}\sum_{b=1}^{2}\sum_{l=1}^{n} X_{khajibl} \leq 1 \;\; \forall a = 1,2; j = 1,2,\dots,n \tag{5}$$
$$\sum_{h=1}^{m}\sum_{i=1}^{m}\sum_{b=1}^{2}\sum_{l=1}^{n} X_{kh20ibl} \leq 1 \;\; \forall k = 1,2,\dots w \tag{6}$$
$$\sum_{h=1}^{m}\sum_{i=1}^{m}\sum_{b=1}^{2}\sum_{l=1}^{n} X_{kh10ibl} = 0 \tag{7}$$
$$\sum_{k=1}^{w}\sum_{h=1}^{m}\sum_{a=1}^{2}\sum_{j=0}^{n} X_{khaji1l} - \sum_{q=1}^{w}\sum_{g=1}^{m}\sum_{c=1}^{2}\sum_{f=0}^{n} X_{qgcfi2l} = 0 \quad \forall i = 1,2,\dots,m; l = 1,2,\dots,n \tag{8}$$
$$\sum_{h=1}^{m}\sum_{a=1}^{2}\sum_{j=0}^{n} X_{khajibl} \geq \sum_{g=1}^{m}\sum_{c=1}^{2}\sum_{f=0}^{n} X_{kiblgcf}$$
$$\forall k = 1,2,\dots w;\; \forall i = 1,2,\dots,m;\; \forall b = 1,2;\; l = 1,2,\dots,n \tag{9}$$
$$O_{bl}^c - T_{bl}^c \geq \sum_{k=1}^{w}\sum_{h=1}^{m}\sum_{a=1}^{2}\sum_{j=1}^{n}\sum_{i=1}^{m} O_{kbl}.X_{khajibl}$$
$$b = 1,2;\; \forall l = 1,2,\dots,n \tag{10}$$
$$T_{bl}^c - O_{aj}^c \geq \sum_{h=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{w} T_{hi}.X_{khajibl} - B.(1 - \sum_{h=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{w} X_{khajibl}) \;\; \forall b = 1,2;\; l = 1,2,\dots,n \;\; \forall a = 1,2; j = 1,2,\dots,n \tag{11}$$
$$O_{2l}^c - P_l^c \geq \sum_{k=1}^{w}\sum_{h=1}^{m}\sum_{a=1}^{2}\sum_{j=i}^{n}\sum_{i=1}^{m} O_{k2l}.X_{khaji2l} \;\; \forall l = 1,2,\dots,n \quad k = 1,2,\dots w \; l = 1,2,\dots,n \tag{12}$$
$$P_l^c - O_{1l}^c \geq P_l \;\; \forall l = 1,2,\dots,n \tag{13}$$
$$\begin{cases} O_{1l}^c - O_{2f}^c \geq \sum_{k=1}^{w}\sum_{h=1}^{m}\sum_{a=1}^{2}\sum_{j=0}^{n} O_{k1l}.X_{khaji1l} \\ -B.\left(2 - \sum_{k=1}^{w}\sum_{h=1}^{m}\sum_{a=1}^{2}\sum_{j=0}^{n}(X_{khaji1l} + X_{khaji1f}) + Q_{fl}\right) \\ O_{1f}^c - O_{2l}^c \geq \sum_{k=1}^{w}\sum_{h=1}^{m}\sum_{a=1}^{2}\sum_{j=0}^{n} O_{k1f}.X_{khaji1l} \\ -B.\left(2 - \sum_{k=1}^{w}\sum_{h=1}^{m}\sum_{a=1}^{2}\sum_{j=0}^{n}(X_{khaji1l} + X_{khaji1f}) + 1 - Q_{fl}\right) \end{cases}$$
$$\forall i = 1,2,\dots,m; k = 1,2,\dots,w;\; f = 1,2,\dots,n; l = f+1, f+2,\dots,n \tag{14}$$
$$O_{20}^c = 0 \tag{15}$$
$$C_{max} \geq O_{2l}^c \;\; \forall l \in J \tag{16}$$
$$NW_k = \sum_{h=1}^{m}\sum_{a=1}^{2}\sum_{j=0}^{n}\sum_{i=1}^{m}\sum_{b=1}^{2}\sum_{l=1}^{m} x_{khajibl}.(O_{kbl} + T_{hi}) \, \forall k = 1,2,\dots,w \tag{17}$$
$$NW_{max} \geq NW_k \quad \forall k = 1,2,\dots,w \tag{18}$$
$$X_{khajibl} \in \{0;1\} \;\forall k = 1,2,\dots w; h = 1,2,\dots,m; a = 1,2;$$
$$j = 0,1,2,\dots,n;\; i = 1,2,\dots,m;\; b = 1,2;\; l = 1,2,\dots,n \tag{19}$$
$$Q_{fl} \in \{0;1\} \;\forall f = 1,2,\dots,n; l = f+1, f+2,\dots,n \tag{20}$$

The mathematical model with the objective function to minimize *makespan* is represented in Eq (3). Eq (4) ensures that only one activity with one machine and operator is allowed for a specific job. Eq (5) ensures that for each specific activity of one job, there is only one preceding activity from another job performed by the same operator. Eq (6) specifies that each operator starts their activities with the unloading activity (index 2) of job 0 (index 0). Eq (7) ensures that job 0 (index 0) does not have a setup activity (index 1). Eq (8) ensures that a specific job only has one setup activity and one unloading activity on the same machine. Eq (9) ensures that each operator follows a feasible activity sequence; activity c of job f precedes activity b of job l, which follows activity a of job j. Eq (10) states that the difference between the completion time of activity b of job l and the moving completion time (when operator k moves to perform activity b of job l) is equal to the time taken by operator k to complete activity b of job l. Eq (11) states that the difference between the moving time (when operator k moves to perform activity b of job l) and the completion time of the previous activity (activity a of job j) is equal to the moving time to activity b of job l. Eq (12) ensures that operator k performs the unloading activity (index 2) of job l only after the machining time for the job is completed. Eq (13) ensures that the machining process starts immediately after the setup activity. Eq (14) includes two constraints for using the same machine, ensuring that the setup activity for job l occurs only after the unloading activity for job f has finished. Eq (15) forces the unloading activity of job 0 to have no duration. Eq (16) calculates the *makespan*. Eq (17) and (18) calculate the total and busiest non-waiting times, respectively. Eq (19) and (20) determine feasible binary variables.

2. Workload Smoothness Index (WSI)

The second objective function is the Workload Smoothness Index (WSI), which measures workload imbalance. It is adapted from the Smoothness Index (SI) used in assembly line balancing problems [19]. WSI is calculated based on each operator's non-waiting time, which includes setup, unloading, and moving activities [19]. The WSI is calculated using the squared deviations between the busiest operator's non-waiting time and the non-waiting time of each operator during a production cycle.

The WSI mathematical model, developed by Akbar and Irohara [19], is non-linear, making it a Mixed-Integer Non-Linear Programming (MINLP) model. Two decision variables are added to the WSI model to achieve the second objective function.

Decision Variables

NW_k = non-waiting time for operator k

NW_{max} = maximum non-waiting time among all operators.

$$\mathrm{WSI}=\sqrt{\textstyle\sum_{k}^{w}(\mathrm{NW_k}\text{-}\mathrm{NW_{max}})^2} \tag{21}$$

Results and Discussions

The mathematical model was developed and executed using Gurobi software across various cases. These cases are labeled in the format $n \times m \times w$, where n represents the total number of jobs, m is the number of machines used, and w is the number of operators assigned. The data processing focuses on finding values to minimize both the first objective function (*makespan*) and the second objective function (operator workload balance). Achieving the *makespan* value is accomplished using a Mixed-Integer Linear Programming (MILP) optimization strategy, which minimizes the *makespan*. Additionally, MILP calculates the differences in operator workload. To balance the workload and achieve a Workload Smoothness Index (WSI) value lower than that from the MILP model, a Mixed-Integer Quadratic Programming (MIQP) optimization strategy is used.

Mixed-Integer Quadratic Programming (MIQP) addresses mathematical optimization problems with quadratic objective functions. The WSI mathematical model is quadratic and squared to fit the MIQP model and Gurobi software's module. The mathematical model for obtaining WSI is:

$$WSI^2 = \textstyle\sum_{k}^{w}(NW_k - NW_{max})^2 \tag{22}$$

Equation (22) is the objective function to balance the operator workload. The MIQP model aims to provide the best *makespan*; this value can be equal to the *makespan* value obtained from the MILP model. To maintain the *makespan* values achieved from the MILP model, an additional constraint is added: the maximum allowable *makespan*, which is the *makespan* value from the MILP model. The *makespan* value in the MIQP model must be less than or equal to this maximum allowable *makespan*. The mathematical model used is:

$$C_{max} \leq C_{max}^{lim} \tag{23}$$

The MIQP results in a WSI value lower than that from the previous model, adjusted to the allowed *makespan*. An additional constraint is introduced to obtain a WSI close to zero, specifying a maximum allowable WSI value. This is achieved using a Mixed-Integer Quadratically-Constrained Programming (MIQCP) optimization strategy. According to this strategy, there is a constraint on the maximum allowable WSI squared value. The results of the MIQCP are *makespan* and WSI values that are less than or equal to the allowed WSI.

$$\mathrm{WSI}^2 \leq WSI_{lim}^2 \tag{24}$$

The MIQCP model results adjust the *makespan* minimization to accommodate the maximum allowable WSI squared value. Three optimization strategies are executed sequentially to obtain the values for the first and second objective functions. Gurobi software (as a solver) is run using different input data sets, each providing a specific result. The obtained results are the *makespan* and WSI, which are compared and analyzed. Additionally, the gap between each result is assessed to evaluate how close the solution found by the solver is to the optimal solution of the problem.

Table 2. The results from Gurobi Software

Cae name	*Solver Setting and Result 1*							*Solver Setting and Result 2*							*Solver Setting and Result 3*						
$n \times m \times w$	*Setting*			*Solution*			*Run Time (s)*	*Setting*			*Solution*			*Run Time (s)*	*Setting*			*Solution*			*Run Time (s)*
	Problem	C_{max}^{lim}	WSI_{lim}^2	C_{max}	*WSI*	*Gap*		*Problem*	C_{max}^{lim}	WSI_{lim}^2	C_{max}	*WSI*	*Gap*		*Problem*	C_{max}^{lim}	WSI_{lim}^2	C_{max}	*WI*	*Gap*	
	MILP	-	-	160.5	61.75	0%	1	MILP	-	-	176.25	56.5	0%	1	MILP	-	-	154.25	36.25	0%	5
4 × 3 × 2	MIQP	160.5	-	160.5	29.75	0%	0	MIQP	176.25	-	176.25	38.5	0%	0	MIQP	154.25	-	154.25	36.25	0%	15
	MIQCP	-	4	290	0.25	100%	600	MIQCP	-	4	202	0.5	100%	600	MIQCP	-	4	299	0.25	100%	600
	MILP	-	-	142.5	14.25	0%	6	MILP	-	-	167.25	9.5	0%	6	MILP	-	-	137	25.25	0%	1
4 × 4 × 2	MIQP	142.5	-	142.5	13.75	0%	1	MIQP	167.25	-	167.25	0.5	0%	1	MIQP	137	-	137	25.25	0%	0
	MIQCP	-	4	311	0.25	100%	600	MIQCP	-	4	456	0.5	100%	600	MIQCP	-	4	271	0.25	100%	600
	MILP	-	-	135	49.45	0%	0	MILP	-	-	139.5	96.47	0%	0	MILP	-	-	122.75	7.28	0%	1
4 × 4 × 3	MIQP	135	-	135	1.6	0%	0	MIQP	139.5	-	139.5	38.27	0%	0	MIQP	122.75		122.75	5.2	0%	5
	MIQCP	-	4	177	0	0%	600	MIQCP	-	4	295	0.25	100%	600	MIQCP	-	4	343	0.25	100%	600
	MILP	-	-	198.25	87	0%	15	MILP	-	-	196.5	11	0%	15	MILP	-	-	208	0.25	0%	15
5 × 3 × 2	MIQP	198.25	-	198.25	87	0%	11	MIQP	196.5	-	196.5	8	0%	11	MIQP	208	-	208	0.25	0%	20
	MIQCP	-	4	215	0	0%	1	MIQCP	-	4	215	0	0%	1	MIQCP	-	4	434	0.25	100%	600
	MILP	-	-	175.5	15	0%	20	MILP	-	-	170.25	28	0%	20	MILP	-	-	178.5	20.75	0%	45
5 × 4 × 2	MIQP	175.5	-	175.5	6	0%	20	MIQP	170.25	-	170.25	3	0%	20	MIQP	178.5		178.5	8.5	0%	71
	MIQCP	-	4	220	0	0%	1	MIQCP	-	4	201	0	0%	1	MIQCP	-	4	384	0.25	100%	600
	MILP	-	-	150.25	54.22	0%	50	MILP	-	-	162	72.15	0%	50	MILP	-	-	154.25	69.9	0%	25
5 × 4 × 3	MIQP	150.25	-	150.25	34	0%	85	MIQP	162	-	162	0.5	0%	85	MIQP	154.25	-	154.25	27.21	0%	90
	MIQCP	-	4	296	0	0%	600	MIQCP	-	4	480	0.5	100%	600	MIQCP	-	4	217	0	0%	66
	MILP	-	-	226.5	64.25	40,33%	535	MILP	-	-	228.25	92.5	41,18	535	MILP	-	-	248	75.5	2,1 %	600
6 × 3 × 2	MIQP	226.5	-	226.5	64.25	0%	600	MIQP	228.25	-	228.25	80.5	100%	600	MIQP	248	-	248	33.5	100%	600
	MIQCP	-	4	266	0	0%	600	MIQCP	-	4	595	0.5	100%	600	MIQCP	-	4	493	0.5	100%	600
	MILP	-	-	201.5	18.75	32,36%	600	MILP	-	-	198.75	9.5	100%	600	MILP	-	-	214.5	11.5	25,64%	285
6 × 4 × 2	MIQP	201.5	-	201.5	18.75	100%	600	MIQP	198.75	-	198.75	11.5	100%	600	MIQP	214.5	-	214.5	11.5	100%	600
	MIQCP	-	4	538	0.25	11,35%	600	MIQCP	-	4	531	0.5	100%	600	MIQCP	-	4	451	0.5	100%	600
	MILP	-	-	186.75	85.85	27,71%	600	MILP	-	-	185.75	73.41	27,72%	600	MILP	-	-	200	41.4	32,27%	600
6 × 4 × 3	MIQP	186.75	-	186.75	0	0%	600	MIQP	185.75	-	-	-	-	Too short	MIQP	200	-	200	29	100%	600
	MIQCP	-	4	311	0	0%	293	MIQCP	-	4	242	0	0%	293	MIQCP	-	4	442	0	100%	600
	MILP	-	-	235.25	12.75	40%	600	MILP	-	-	231.75	28.75	42%	600	MILP	-	-	244	17.75	43,13 %	600
7 × 4 × 2	MIQP		-	-	-	-	Too short	MIQP	231.75	-	231.75	32.75	100%	600	MIQP	-	-	-	-	-	Too short
	MIQCP	-	4	622	0.25	100%	600	MIQCP	-	4	523	0	100%	600	MIQCP	-	4	326	0	0%	109
	MILP	-	-	221.75	22.25	39,12%	600	MILP	-	-	205.5	32.57	34,67%	600	MILP	-	-	288.75	30.37	41,10%	595
7 × 4 × 3	MIQP	221.75	-	221.75	3.25	100%	600	MIQP	-	-			100%	600	MIQP	288.75		288.75	11.33	100%	600
	MIQCP	-	4	759	0.25	100%	600	MIQCP	-	4	606	0.25	100%	600	MIQCP	-	4	523	0.25	100%	600
	MILP	-	-	270.5	58.25	48,44%	479	MILP	-	-	288.75	20.5	52,8%	479	MILP	-	-	288.75	20.5	53,50%	212
8 × 4 × 2	MIQP		-	-	-	-	Too short	MIQP	288.75	-	-	-	-	Too short	MIQP			-	-	-	Too short
	MIQCP	-	4	707	0.25	100%	600	MIQCP	-	4	493	0.25	100%	600	MIQCP	-	4	530	0.5	83,59 %	600
	MILP	-	-	241	102.61	43,98%	600	MILP	-	-	222	104.52	39,52%	600	MILP	-	-	246	49.25	46,44%	600
8 × 4 × 3	MIQP		-	-	-	-	Too short	MIQP		-	-	-	-	Too short	MIQP			-	-	-	Too short
	MIQCP	-	4	293	0	0%	303	MIQCP	-	4	349	0	0%	303	MIQCP	-	4	557	0	0%	135
	MILP	-	-	306.5	12.5	55,95%	600	MILP	-	-	303.5	16	55,76%	600	MILP	-	-	324.25	6.5	58, 13%	596
9 × 4 × 2	MIQP	306.5		306.5	4.25	100%	600	MIQP	303.5		303.5	2	100%	600	MIQP	324.25		324.25	1.5	100%	600
	MIQCP	-	4	726	0.25	100%	194	MIQCP	-	4	369	0	0%	194	MIQCP	-	4	654	0.5	100%	600
	MILP	-	-	264.75	88.27	49%	600	MILP	-	-	254	93.44	47,14%	600	MILP	-	-	266.5	80.3	50,56%	597
9 × 4 × 3	MIQP	264.75	-	264.75	62.56	100%		MIQP		-	-	-	-	Too short	MIQP	266.5		-	-	-	Too short
	MIQCP	-	4	585	0.5	100%	600	MIQCP	-	4	630	0.25	100%	600	MIQCP	-	4	676	0.35	100%	600
	MILP	-	-	321.75	14.75	56,63%	600	MILP	-	-	380.25	66.75	64,69%		MILP	-	-	347.75	41.25	62,11%	600
10 × 4 × 2	MIQP	321.75		-	-	-	Too short	MIQP	380.25		380.25	0.25	100%	600	MIQP	347.75		347.75	2.4	100%	600
	MIQCP	-	4	601	0.25	100%	600	MIQCP	-	4	596	0.25	100%	600	MIQCP	-	4	750	0.25	100%	600
	MILP	-	-	266.75	82.299	49,39%	600	MILP	-	-	292	29.66	54,02%	600	MILP	-	-	285.5	43.62	53,85%	600
10 × 4 × 3	MILP	-	-	-	-	-	Too short	MIQP		-	-	-	-	Too short	MIQP	-	-	285.5	43.62	100%	600
	MIQP	160.5	4	160.5	29.75	0%	600	MIQCP	-	4	637	1.03	100%	600	MIQCP	-	4	716	0.55	100%	600

Note:
Small Cases: 4 × 3 × 2, 4 × 4 × 2, 4 × 4 × 3, 5 × 3 × 2, 5 × 4 × 2, 5 × 4 × 3
Medium Cases: 6 × 4 × 2, 6 × 4 × 3, 7 × 4 × 2, 7 × 4 × 3, 8 × 4 × 2, 8 × 4 × 3, 9 × 4 × 2, 9 × 4 × 3, 10 × 4 × 2, 10 × 4 × 3

The MILP model was solved optimally (0% gap) by the solver, providing the best *makespan,* especially in minor cases such as 4 × 3 × 2, 4 × 4 × 2, 4 × 4 × 3, 5 × 3 × 2, 5 × 4 × 2, and 5 × 4 × 3. As previously mentioned, the solver cannot guarantee a good WSI since it is not included in the objective function, as shown by results such as a WSI of 61.75 for case 4 × 3 × 2 and 14.25 for case 4 × 4 × 2. A large WSI indicates a significant workload gap for each operator, in medium cases, such as 6 × 3 × 2, the gap rises to 40.33% (result 1), 41.18% (result 2), and 42.1% (result 3). This increasing gap suggests that the solver could not find the optimal solution, as the branch and bound search should continue. However, the solver's running time was limited to shorten data processing time. Extending the running time could decrease the gap, but it cannot be guaranteed that the solver will reach the optimal solution. Overall, the *makespans* from the MILP model have a relatively low range (between result 1 and others) because the average values of each input data set are nearly the same.

Like the MILP model, the MIQP model can also be solved optimally (0% gap) in minor cases. The critical difference is that the WSI value in the MIQP model is lower than in the MILP model. This difference in WSI occurs after applying Eq (22) to obtain the optimal solution with the squared element. In medium cases, such as 7 × 4 × 3, the gap and runtime increase, with gaps of 100% in results 1 and 3. This increase is due to the model's complexity, which complicates the search for the optimal solution. This is compounded by the additional mathematical model, making it more complex than the previous one. The results show that the MIQP model has some cases where a feasible solution cannot be obtained without extended running time.

The MIQCP model includes a constraint that allows the WSI (Eq 24) to decrease workload imbalance. The results show that the WSI in the MIQCP model is less than that in both the MILP and MIQP models. However, the *makespan* in the MIQCP model is higher than in the two previous models. The MIQCP model adjusts the resulting *makespan* to balance the operator workload. Sometimes, the MIQCP model increases the gap (100%) due to minimal movement in the solver's bounds; the solution barely changes, and the best bound remains at 0. The increased complexity of the model contributes to this widening gap, making the solution far from optimal. Gurobi's output in the resolve section shows the total number of constraints (rows), variables (columns), and iterations, reflecting the complexity of the mathematical model.

Among the three MIQCP results with different input data sets, some cases achieve an optimal gap (0%), such as 4 × 4 × 3 and 5 × 3 × 2 with 0% gaps in results 1 and 2. However, result 3 shows a 100% gap. Variations in input data can affect the overall optimization problem, leading to different results, calculation complexities, and solver gaps. Each data set has unique characteristics that result in varying solutions (e.g., Gantt chart scheduling, *makespan*, and WSI) and solution spaces.

The results are depicted in three Gantt charts illustrating machine and operator activities and each operator's workload. The Gantt charts for case 5 × 4 × 3 are used to determine the differences between the optimization strategies.

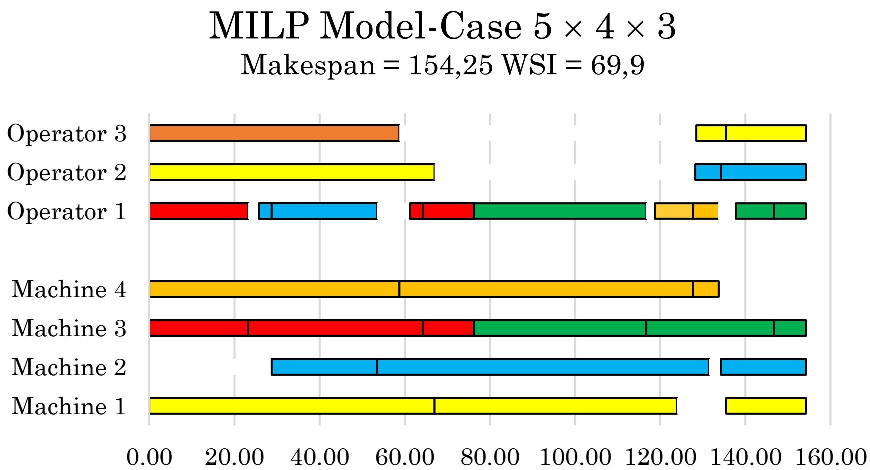

Figure 1. Gantt chart of MILP model

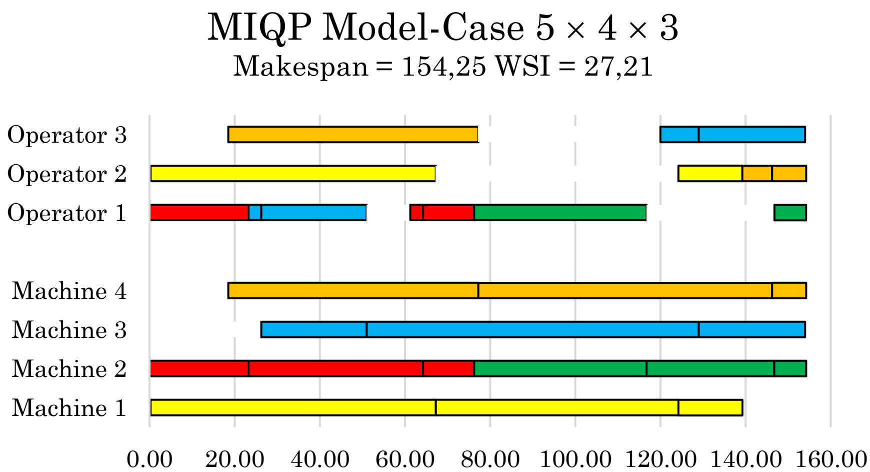

Figure 2. Gantt chart of MIQP model

The Gantt charts reveal significant differences between the three optimization strategies. For five jobs, four machines, and two operators with different skill levels, the MILP model provides the best *makespan* but indicates a large WSI, signifying workload imbalance, with operator 1 busier than operators 2 and 3. The MIQP model, with an allowed *makespan*, can minimize WSI after applying Eq (22). The MIQCP Gantt chart shows balanced operator workloads, although the resulting *makespan* is higher.

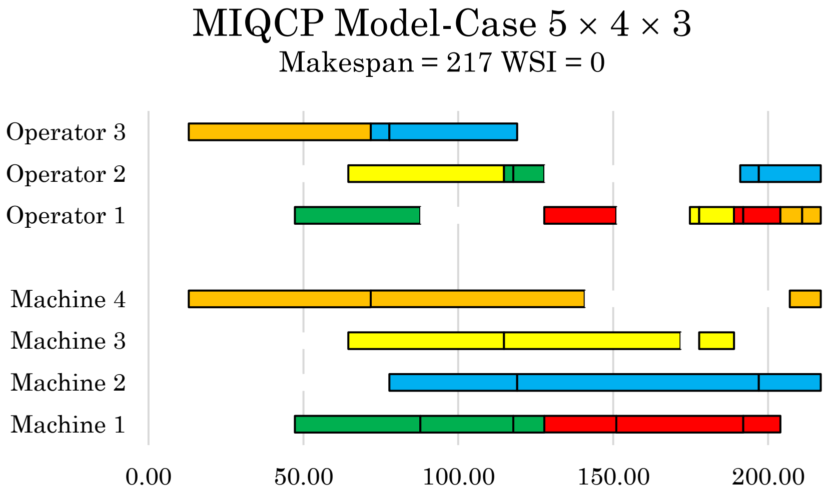

Figure 3. Gantt chart of MIQCP model

For more details, Table 3 presents non-waiting times, idle times, and the number of activities performed by each operator.

Table 3. Number of activities, Non-waiting Time, *Makespan*, and Idle Time from the operators

Model	Operator	Operator 1	Operator 2	Operator 3
MILP	Number of activities	6 activities	2 activities	2 activities
	Non-waiting Time	138	93	84.5
	Makepan	154.25	154.25	154.25
	Idle Time	16.25	61.25	69.75
MIQP	Number of activities	5 activities	3 activities	2 activities
	Non-waiting Time	114	97	92.75
	Makepan	154.25	154.25	154
	Idle Time	40.25	57.25	61.25
MIQCP	Number of activities	5 activities	3 activities	2 activities
	Non-waiting Time	106	106	106
	Makepan	217	216.75	119
	Idle Time	110	110.75	13

Note:
The operator's activities include setup and unloading

The Gantt chart and Table 3 aim to illustrate how differences in skill levels between operators influence the performance values of different operators. Additionally, Table 3 shows the variations in the output obtained from the MILP, MIQP, and MIQCP models. The table includes three operators with varying skill levels: above average, average, and below average. In the MILP model, operator 1 performs more tasks and has a higher non-waiting time than other operators, with relatively low idle time. Meanwhile, operators 2 and 3 have higher idle times. Since the objective function in MILP is to minimize the *makespan,* the model tends to assign more tasks to operator 1, who has a higher skill level, as evidenced by the number of activities performed—conversely, operators 2 and 3 exhibit lower efficiency with higher idle times.

In the MIQP model, workload balancing becomes an objective function calculated from the WSI value, with an additional constraint of the allowable *makespan*. As a result, the WSI value decreases more than in the MILP model. The output shows an increase in efficiency for operators 1 and 2: operator 1 performs 5 activities with a non-waiting time of 144, while operator 2 performs 3 activities with a non-waiting time of 93. However, operator 3 shows only a slight improvement in efficiency and reduced idle time.

In the MIQCP model, the WSI value serves as a constraint to achieve a balanced distribution of non-waiting time, with the objective function focusing on minimizing the *makespan*. The results indicate that operators 1, 2, and 3 have a non-waiting time of 106. However, operators 1 and 2 still have high idle times, while operator 3 has significantly lower idle time. Table 3 shows that operators with higher skill levels complete more activities, but operators 2 and 3 have fewer assignments. This occurs because operator 1 performs activities faster, allowing them to be assigned to subsequent tasks more quickly.

Conclusions

This research developed the MTSSDRC model with two objective functions and introduced a new consideration: differences in operator skill levels. The complexity of the problem increases as each operator has different skill levels, necessitating a balance between minimizing the *makespan* and balancing the operator's workload in each production cycle. This issue is common in real-world systems, where finding industries with operators of uniform skill levels is challenging. This scenario mirrors many industries, such as aircraft manufacturing, where operators supervise multiple machines simultaneously, each with varying skills.

The developed mathematical model was executed using three sequential optimization strategies: MILP, MIQP, and MIQCP. These techniques allocate operators with various skill levels according to available jobs. The results of these techniques demonstrate that skill level differences can impact efficiency and workload balance. Typically, operators with higher skills are assigned more activities, while those with lower skills receive fewer tasks. By incorporating these techniques into operational strategies, companies can enhance the skills of their operators to improve efficiency in aircraft component production.

However, this research has limitations. The solver and model used are effective only for small and medium cases, with some solutions deviating significantly from the optimal. To address this, a metaheuristic algorithm is proposed to solve the MTSSDRC scheduling problem. This algorithm is well-suited for finding optimal solutions for all cases within shorter running times. Additionally, metaheuristic algorithms can efficiently handle complex problems and significant cases.

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