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Integrated Optimization of Heterogeneous Fleet Deployment, Sailing Speed, and Bunkering Strategy Considering Adaptive Safety Stock

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Abstract: Logistics cost inefficiencies often stem from fragmented operational policies. Volatile global fuel prices and unpredictable maritime schedules further complicate matters. Traditional isolated optimization methods frequently fail to ensure supply chain resilience. This study addresses these limitations by developing a Mixed-Integer Linear Programming (MILP) model. The model simultaneously integrates three core strategic decisions: heterogeneous fleet deployment, sailing speed optimization, and bunkering strategy. Inventory thresholds are dynamically adjusted based on real-time sailing conditions and port-to-port consumption rates, moving beyond static buffer assumptions. This model incorporates an adaptive stock mechanism to mitigate energy supply uncertainties at transit ports while minimizing total costs, which diverges from conventional approaches. The mathematical formulation is designed to minimize total operating expenses while accounting for technical constraints, such as fixed time windows and fluctuating cargo capacities. Optimization results show that integrating these variables effectively reduces cost inefficiencies. Quantitatively, the Proposed Scenario reduced Total Cost by 18.89%, saving USD 191,555 per service cycle compared to the Existing Scenario. The integrated approach uncovers a significant trade-off between speed reduction and inventory holding costs, identifying a more balanced operational equilibrium than previous models. The findings demonstrate that applying adaptive safety stock enhances the robustness of the bunkering strategy by aligning minimum inventory levels with fuel consumption across segments between bunkering ports. This study contributes to maritime management theory by synchronizing adaptive fuel inventory management with vessel deployment and speed optimization. There are practical implications for designing more resilient and cost-effective shipping strategies. Finally, this framework serves as a precursor tool for shipping liners to maintain service reliability while navigating the complexities of modern maritime logistics.

Keywords: Fleet deployment, bunkering strategies, liner shipping, fuel consumption, speed optimization, mixed-integer linear programming.

Introduction

As the global economy accelerates, maritime transportation has has become the backbone of international trade [1]. According to The International Chamber of Shipping (ICS), the industry carries 11 billion tons of goods annually [2], accounting for over 80% of total global trade volume. However, managing this massive scale introduces significant economic and logistical challenges, particularly due to high operational costs and supply chain uncertainties. This dynamic is highly relevant to Indonesia, an archipelagic nation where water constitutes over 50% of its territory. Strategically located between the Indian and Pacific Oceans, approximately 40% of global trade passes through Indonesian waters [3].

Compared to regional counterparts such as Singapore, Thailand, and Malaysia, Indonesian ports currently underperform across almost every dimension. Service quality and operational efficiency are especially lacking. A primary reason for this disparity is economic: The maritime sector accounts for over 80% of GDP in those neighboring countries, compared to less than 20% in Indonesia. Furthermore, domestic maritime trade remains

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concentrated on Java Island. Eastern Indonesia is underdeveloped due to vast distances and low trade volumes, leading to much higher cargo transportation costs in the East than in the West. To help correct this imbalance, the Indonesian government introduced the Tol Laut (Sea Tollway) initiative in 2014. This policy connects 24 strategic ports across 18 routes to enhance national connectivity and optimize cost efficiency [3].

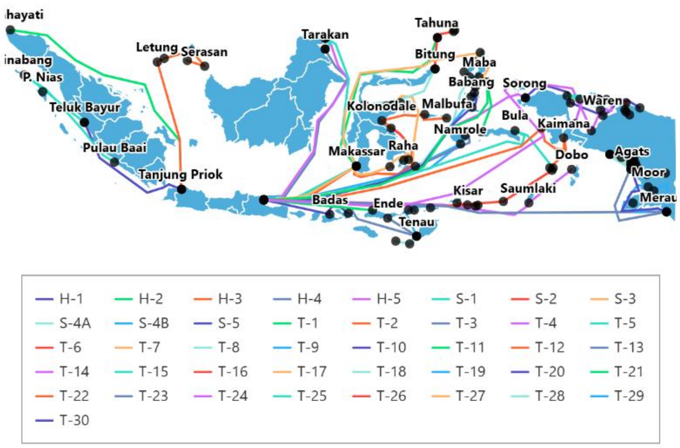

Figure 1. Sea Tollway routes [4]

Operating cargo ships involves significant costs, with fuel accounting for 30-60% of total vessel operating costs. For shipping companies, managing bunkering strategies and vessel speeds is vital. These decisions reduce operational costs and enhance market competitiveness [5]. Managing inventory effectively involves strategic replenishment that exploits price differences while strictly respecting storage capacities [6]. Our framework adopts this perspective by determining optimal bunkering volumes at ports with the most competitive fuel rates. The goal is to minimize total operating costs. While bunkering research is extensive, the application of formal inventory policies, such as the (s, S) policy, remains rare in operational models [7]. Despite a large body of research on liner shipping optimization, a significant gap remains in integrating vessel operations with inventory reliability. Most studies approach fleet deployment, sailing speed, and bunkering as cost-minimization problems for carriers. They often assume deterministic arrivals and manage inventory levels at destination ports separately. As a result, research that explicitly links maritime operational decisions to adaptive safety stock is lacking.

To address this gap, this study makes two contributions. First, we develop a novel Mixed-Integer Linear Programming (MILP) model. It synchronizes heterogeneous fleet deployment, sailing speed, and bunkering strategy with an endogenous adaptive safety stock. Unlike existing research, our model treats safety stock as a dynamic decision variable tied to fuel consumption per voyage leg. This approach captures critical interdependencies between energy efficiency and supply chain reliability. Second, the model is validated through a real-world case study with a leading Indonesian Sea Tollway operator. We utilize empirical data to provide actionable insights for balancing operational costs and service resilience across Indonesia's diverse archipelago.

By addressing these contributions, our study not only advances the quantitative optimization of maritime logistics within the Sea Tollway network but also underscores the critical importance of integrating operational decisions with adaptive safety stock mechanisms. This integrated approach provides a robust framework for maritime operators to balance minimizing operational costs with ensuring high supply chain reliability in geographically challenging, volatile regions.

Literature Review

Liner shipping carries most containerized goods through fixed schedules and routine services [8]. Conventionally, these lines operate on cyclic routes, where vessels follow a predetermined and repetitive path [9]. A fundamental decision-making problem in liner shipping is the fleet deployment problem [10]. Fleet deployment is a strategic issue that determines the type and number of vessels on each service route [11]. It involves allocating available vessels to service routes to satisfy demand within a specific time horizon at the lowest possible cost [3], [12]. According to [13], fleet deployment involves determining the appropriate vessel types for each route. The problem concerns a company operating a set of fixed service routes, where each vessel has distinct carrying capacity and fuel-consumption characteristics [14].

In shipping operations, speed-optimization policies play a critical role in cost management frameworks. Practical experience indicates that higher vessel speeds lead to greater fuel consumption [15]. While lower speeds (slow steaming) can significantly reduce fuel consumption, they may disrupt delivery schedules and violate arrival time windows [6]. Consequently, it is essential to integrate vessel speed optimization, fuel consumption, and vessel type selection [16]. Since the relationship between vessel speed and fuel consumption is non-linear, adjusting speed for each voyage leg can significantly reduce total fuel consumption [6], [17].

Research on bunkering strategies focuses on selecting optimal ports and refueling volumes to minimize total procurement costs [18]. Among various cost components, fuel remains the most dominant, accounting for up to 60% of total vessel operating costs [19]. In liner shipping, bunkering strategies involve selecting ports along a fixed route to exploit regional price differentials [20]. Recently, researchers have begun integrating speed optimization and bunkering simultaneously to provide more efficient operational strategies [21]. This paper contributes by developing an integrated model for fleet deployment, speed, and bunkering optimization, and by implementing adaptive stock-keeping policies to minimize total operational costs.

Table 1. Research literature

Literature	Decision-Making Content			Safety Stock Policies	
	Fleet Deployment	**Speed Optimization**	**Bunkering**	**Static**	**Adaptive**
Lashgari et al. [22]	✓	✓	✓	✓	-
Zhuge et al. [23]	✓	✓	-	-	-
Wu et al. [24]	✓	✓	-	-	-
Lin et al. [25]	✓	✓	-	-	-
He et al. [26]	-	✓	✓	-	-
Wang et al. [27]	-	✓	✓	-	-
Xing et al. [28]	-	✓	-	-	-
Zhao et al. [29]	✓	✓	✓	-	-
Proposed Research	✓	✓	✓	-	✓

Methods

Research Design

This study adopts a quantitative, optimization-based research design to develop and evaluate an integrated bunkering planning model. The operational problem is formulated as a Mixed-Integer Linear Programming (MILP) model that jointly determines vessel deployment, discrete sailing-speed selection per leg, and bunkering quantities across eligible ports, while enforcing schedule feasibility through time-related constraints. The MILP is implemented in Python using CPLEX and solved exactly with IBM ILOG CPLEX Optimization Studio (v22.1.2) via a branch-and-cut framework.

The research procedure consists of (i) preprocessing the case data and deriving leg-based parameters, (ii) constructing and solving the MILP, and (iii) post-processing the solution into operational decisions (deployment, speed profile, bunkering plan, fuel inventory trajectory, and lateness). In the case implementation, most input data are treated as given based on company records and operational datasets; however, several inputs are assumed due to data limitations, particularly selected cost parameters and the leg-level cargo (on-board load) profile constructed during preprocessing. The proposed adaptive safety stock policy is evaluated through computational experiments by comparing it against a static safety stock baseline under the same case setting.

Case Setting and Data

This study is based on the operational context of Liner Shipping Company, a scheduled maritime logistics operator in Indonesia. The service network is modeled as five fixed service routes (*trayek*), each represented as a closed loop with a predetermined port-call sequence. Across the five routes, the network visits 22 distinct ports, with Tanjung Perak and Tanjung Priok serving as the main hubs/origin–return ports, depending on the route. The total sailing distance per route ranges from 1,846 NM (Route 2) to 2,625 NM (Route 4), reflecting heterogeneous route lengths and operational exposure.

Input Data Components

The case data used in the model are organized into four main groups.
Fleet data (candidate vessels): The candidate fleet consists of five general cargo vessels with heterogeneous technical and cost characteristics. Vessel capacity ranges from 115–290 TEU, with operational non-fuel cost of approximately 70.78–93.41 USD/hour. Fuel tank capacity (in the dataset) ranges from 304–841 MT, and each vessel has a distinct fuel consumption profile linked to speed.

Speed and fuel consumption data: Sailing speed is represented using three discrete levels (minimum, average, and maximum) for each vessel, enabling speed-selection decisions at the leg level. The observed speed range across vessels spans approximately 6.4–12.9 knots, with associated fuel consumption rates ranging from 0.282 to 1.968 MT/hour, depending on vessel type and selected speed level.

Route structure and sailing distances: For each route, the ordered port-call sequence is converted into a set of legs (a leg connects two consecutive ports). Sailing distances are provided in nautical miles (NM) for the route/leg representation, serving as the basis for computing sailing time under each speed level.

Cargo/load data (per leg): Cargo is treated as exogenous and represented as the total onboard load per leg (TEU), derived from the company's cargo records for each route. The resulting leg-load profiles vary by route (e.g., Route 4 exhibits the largest leg-load values, reaching 254 TEU on certain legs), and are used to enforce vessel capacity feasibility along the route.

Additionally, bunkering-related inputs include port-level bunkering availability and bunkering price (USD/MT). In this case, 15 ports are identified as bunkering-enabled, with prices ranging from 1,328 to 1,593 USD/MT.

Assumptions

The number of available vessels is fixed, and the technical specifications of each vessel are known. Each vessel departs from its origin port with a given initial fuel inventory level. The service network follows predetermined circular routes. Each port has an arrival time window that must be met; late arrivals are allowed but incur a penalty. Bunkering decisions are made based on the onboard fuel inventory level upon vessel arrival at each port, and fuel replenishment is permitted only at ports where bunkering service is available. Port service time is assumed to be constant (exogenous) and applied in schedule propagation. The operational time limit per leg is defined based on planned sailing time (distance and planned speed) with an additional contingency buffer. It is used to evaluate lateness/penalty within the optimization.

MILP Formulation

This section explains the Mixed-Integer Linear Programming (MILP) formulation that integrates ship assignment decisions, selection of speed levels per leg, and bunkering strategy, while ensuring that cargo capacity limits, fuel tank capacity limits, adaptive safety stock policies, and schedule limits over time per leg are met. Within this research framework, a 'leg' refers to the distance between one port and the next along a shipping route. It serves as the basic unit of analysis for determining operational variables such as sailing speed, transit time, and fuel consumption between two consecutive nodes in the network. The model uses the following notation in Table 2.

Table 2. Mathematical notation

Notation	Descriptions
Sets	
$\boldsymbol{K}$	Set of ships
$\boldsymbol{R}$	Set of routes
$\boldsymbol{P_r}$	Ordered set of ports on route $\boldsymbol{r}$
$\boldsymbol{L_r}$	Set of legs on route $\boldsymbol{r}$, where leg $\boldsymbol{\ell}$ connects port $\boldsymbol{\ell}$ to port $\boldsymbol{\ell+1}$ (leg $\boldsymbol{\ell : \ell \rightarrow \ell+1}$)
$\boldsymbol{S}$	Set of discrete speed levels $\{\mathbf{1, 2, 3}\}$
Parameters	
$\boldsymbol{Cap_k}$	Cargo capacity of ship $\boldsymbol{k}$ (TEU)
$\boldsymbol{Cap_k^{tank}}$	Maximum fuel tank capacity of ship $\boldsymbol{k}$ (MT)

Notation	Descriptions
OC_k	Operational non-fuel cost of ship k (USD/hour)
$V_{k,s}$	Sailing speed of ship k at speed level s (knot, NM/hour)
V_{plan}	Planned speed used for the linearization of sailing time (knot)
$FR_{k,s}$	Fuel consumption rate of ship k at speed level s (MT/hour)
$d_{r,\ell}$	Sailing distance on route r, leg ℓ (NM)
$Req_{r,\ell}$	Required onboard cargo on route r, leg ℓ (TEU)
$ReqSS_{k,r,p}$	Required minimum Safety Stock of ship k on route r at port p (MT)
$A_{r,p}$	Bunkering availability indicator on route r at port p
$price_{r,p}$	Bunker price on route r at port p (USD/MT)
α	Adaptive safety stock factor
h	Fuel holding cost rate (USD/MT/hour)
π	Delay penalty cost (USD/hour)
$serv$	Port service time (hours)
β	Initial fuel ratio relative to tank capacity
$t_{k,r,\ell,s}$	Sailing time of ship k on route r, leg ℓ, at speed level s (hours)
$t_{r,l}^{plan}$	Planned sailing time on route r, leg ℓ (hours)
$cons_{k,r,\ell,s}$	Fuel consumption of ship k on route r, leg ℓ, at speed level s (MT)
$cons_{k,r,p,s}$	Fuel consumption of ship k on route r for the leg departing from port p at speed level s (MT)
$\Delta_{r,\ell}$	Time limit on route r, leg ℓ (hours)
$nb(r,p)$	Index of the next bunkering port after port p on route r
$D_{r,o,d}$	OD demand on route r, from origin o to destination d
Decision Variables	
$x_{k,r}$	1 if ship k is assigned to route r; 0 otherwise
$y_{k,r,\ell,s}$	1 if ship k assigned to route r selects speed level s on leg ℓ; 0 otherwise
$y_{k,r,p,s}$	1 if ship k assigned to route r selects speed level s on the leg departing from port p; 0 otherwise
$T_{r,p}$	Arrival time on route r at port p (hours)
$Late_{r,\ell}$	Delay on route r, leg ℓ (hours)
$b_{k,r,p}$	Bunkering quantity purchased by ship k on route r at port p (MT)
$I_{k,r,p}^{arr}$	Fuel inventory on board of ship k on route r upon arrival at port p (MT)
$I_{k,r,p}^{dep}$	Fuel inventory on board of ship k on route r upon departure from port p (MT)
$SegCons_{k,r,p}$	Total fuel consumption of ship k on route r, from port p to the next bunkering port $nb(r,p)$ (MT)
$LegCons_{k,r,p}$	Fuel consumption of ship k on route r, for the leg from port p to port $p+1$ (MT)
$I_{r,\ell,}^{dep}$	Fuel inventory upon departure on route r, leg ℓ (MT)
$I_{r,\ell+1}^{arr}$	Fuel inventory upon arrival on route r, leg ℓ (MT)
Cost terms	
TC	Total operating cost over one sailing cycle (USD)
C^{fleet}	Fleet operating cost over one sailing cycle (USD)
C^{bunk}	Bunkering cost over one sailing cycle (USD)
C^{hold}	Fuel inventory holding cost over one sailing cyle (USD)
C^{pen}	Delay penalty cost over one sailing cycle (USD)

Mathematics Model

The objective of the model is to minimize the total cost:

$$min\, TC = C^{fleet} + C^{bunk} + C^{hold} + C^{pen} \tag{1}$$

The fleet operation cost is the sum of the total travel time and service time:

$$C^{fleet} = \sum_{k\in K}\sum_{r\in R}\sum_{\ell\in L_r}\sum_{s\in S} OC_k \cdot t_{k,r,\ell,s} \cdot y_{k,r,\ell,s} + \sum_{k\in K}\sum_{r\in R} OC_k \cdot serv \cdot |L_r| \cdot x_{k,r} \tag{2}$$

The bunkering cost is calculated as follows:

$$\boldsymbol{C^{bunk}} = \sum_{k\in K}\sum_{r\in R}\sum_{p\in P_r} \boldsymbol{price_{r,p} \cdot b_{k,r,p}} \tag{3}$$

The holding cost for fuel inventory is calculated as follows:

$$\boldsymbol{C^{hold}} = \sum\nolimits_{r\in R}\sum\nolimits_{\ell\in L_r} \frac{\boldsymbol{h}}{\boldsymbol{2}}\cdot\left(\boldsymbol{I^{dep}_{r,\ell}} + \boldsymbol{I^{arr}_{r,\ell+1}}\right)\cdot \boldsymbol{t_{plan}} \tag{4}$$

The penalty cost for delays per leg is calculated as follows:

$$\boldsymbol{C^{pen}} = \sum_{r\in R}\sum_{\ell\in L_r} \boldsymbol{\pi \cdot Late_{r,\ell}} \tag{5}$$

Subjected to:

$$\sum_{k\in K} \boldsymbol{x_{k,r} = 1} \qquad \forall \boldsymbol{r} \in \boldsymbol{R} \tag{6}$$

$$\sum \boldsymbol{x_{k,r} \le 1} \qquad \forall \boldsymbol{k} \in \boldsymbol{K} \tag{7}$$

$$\sum_{s\in S} \boldsymbol{y_{k,r,\ell,s} = x_{k,r}} \qquad \forall \boldsymbol{k} \in \boldsymbol{K}, \forall \boldsymbol{r} \in \boldsymbol{R}, \forall \boldsymbol{\ell} \in \boldsymbol{L_r} \tag{8}$$

$$\sum_{k\in K} \boldsymbol{Cap_k \cdot x_{k,r} \ge Req_{r,\ell}} \qquad \forall \boldsymbol{r} \in \boldsymbol{R}, \forall \boldsymbol{\ell} \in \boldsymbol{L_r} \tag{9}$$

$$\boldsymbol{T_{r,1} = 0} \qquad \forall \boldsymbol{r} \in \boldsymbol{R} \tag{10}$$

$$\boldsymbol{T_{r,\ell+1} \ge T_{r,\ell} + serv} + \sum_{k\in K}\sum_{s\in S} \boldsymbol{t_{k,r,\ell,s} \cdot y_{k,r,\ell,s}} \qquad \forall \boldsymbol{r} \in \boldsymbol{R}, \forall \boldsymbol{\ell} \in \boldsymbol{L_r} \tag{11}$$

$$\boldsymbol{Late_{r,\ell}} \ge \left(\sum_{k\in K}\sum_{s\in S} \boldsymbol{t_{k,r,\ell,s} \cdot y_{k,r,\ell,s}}\right) - \Delta_{r,\ell} \qquad \forall \boldsymbol{r} \in \boldsymbol{R}, \forall \boldsymbol{\ell} \in \boldsymbol{L_r} \tag{12}$$

$$\boldsymbol{Late_{r,\ell} \ge 0} \qquad \boldsymbol{\ell} \in \boldsymbol{L_r} \tag{13}$$

$$\boldsymbol{I^{arr}_{k,r,1} = \beta Cap^{tank}_k x_{k,r}} \qquad \forall \boldsymbol{k} \in \boldsymbol{K}, \forall \boldsymbol{r} \in \boldsymbol{R} \tag{14}$$

$$\boldsymbol{I^{dep}_{k,r,p} = I^{arr}_{k,r,p} + b_{k,r,p}} \qquad \forall \boldsymbol{k}, \boldsymbol{r}, \forall \boldsymbol{p} \in \boldsymbol{P_r} \tag{15}$$

$$\boldsymbol{I^{arr}_{k,r,\ell+1} = I^{dep}_{k,r,\ell}} - \sum_{s\in S} \boldsymbol{cons_{k,r,\ell,s} \cdot y_{k,r,\ell,s}} \qquad \forall \boldsymbol{k}, \boldsymbol{r}, \forall \boldsymbol{\ell} \in \boldsymbol{L_r} \tag{16}$$

$$\boldsymbol{b_{k,r,p} \le (Cap^{tank}_k - I^{arr}_{k,r,p})\, A_{r,p} x_{k,r}} \qquad \forall \boldsymbol{k}, \boldsymbol{r}, \forall \boldsymbol{p} \in \boldsymbol{P_r} \tag{17}$$

$$\boldsymbol{SegCons_{k,r,p}} = \sum_{\ell=p}^{nb(r,p)-1}\sum_{s\in S} \boldsymbol{cons_{k,r,\ell,s} \cdot y_{k,r,\ell,s}} \qquad \forall \boldsymbol{k}, \boldsymbol{r}, \boldsymbol{p}: \boldsymbol{A_{r,p}} \tag{18}$$

$$\boldsymbol{LegCons_{k,r,p}} = \sum_{s\in S} \boldsymbol{cons_{k,r,p,s} \cdot y_{k,r,p,s}} \qquad \forall \boldsymbol{k} \in \boldsymbol{K}, \forall \boldsymbol{r} \in \boldsymbol{R}, \forall \boldsymbol{p} \in \boldsymbol{P}: \boldsymbol{p+1} \in \boldsymbol{P_r} \tag{19}$$

$$\boldsymbol{ReqSS_{k,r,p}} = \begin{cases} \boldsymbol{\alpha \cdot Legcons_{k,r,p}} & \boldsymbol{if\ nb(r,p) = p+1} \\ \boldsymbol{(1+\alpha)\cdot\left(SegCons_{k,r,p} - Legcons_{k,r,p}\right)} & \boldsymbol{if\ nb(r,p) > p+1} \end{cases} \tag{20}$$

$$\boldsymbol{I^{arr}_{k,r,p} \ge ReqSS_{k,r,p}} \qquad \forall \boldsymbol{k} \in \boldsymbol{K}, \forall \boldsymbol{r} \in \boldsymbol{R}, \forall \boldsymbol{p} \in \boldsymbol{P}: \boldsymbol{p+1} \in \boldsymbol{P_r} \tag{21}$$

$$\boldsymbol{0 \le I^{arr}_{k,r,p} \le Cap^{tank}_k x_{k,r}} \qquad \forall \boldsymbol{k}, \boldsymbol{r}, \boldsymbol{p} \tag{22}$$

$$\mathbf{0} \leq \boldsymbol{I_{k,r,p}^{dep}} \leq \boldsymbol{Cap_k^{tank} x_{k,r}} \qquad \forall \boldsymbol{k, r, p} \tag{23}$$

The objective function, defined in Equation (1), minimizes the total operating cost over one sailing cycle. This total cost is formulated as the sum of four main components: time-based fleet operating cost, bunkering cost, onboard fuel inventory holding cost during sailing, and delay penalty cost with respect to the time limit on each leg.

Equations (2)–(5) further decompose the four cost components in the objective function. Equation (2) calculates fleet operating cost as the vessel's non-fuel operating cost multiplied by the total travel time plus the accumulated port service time. In the context of this study, non-fuel operating costs are defined as the aggregate of all fixed and variable expenses required to maintain and operate a vessel during its voyage, excluding bunker fuel costs. This comprehensive metric encompasses crew wages, administrative overhead, regular maintenance, insurance, provisions, and other daily running costs essential for the vessel's functionality and compliance, measured on a per-time-unit basis. Equation (3) computes bunkering cost as the total fuel purchased at ports where bunkering is conducted, accounting for the bunker price at each port. Equation (4) computes holding cost as the cost of carrying onboard fuel inventory during sailing; to preserve model linearity, the holding cost formulation uses the planned sailing time. Equation (5) computes the delay penalty cost as the per-hour cost of delay triggered by the leg-level lateness variable.

Constraints (6)-(7) arrange fleet deployment (ship assignment). Constraint (6) ensures that each route is served by exactly one ship (one-route-one vessel), while constraint (7) ensures that no ship serves multiple routes simultaneously.

Constraint (8) arrange the selection of speed levels per leg. Constraint (8) ensures that if a ship is assigned to a route $\boldsymbol{r}$, it must select one speed level from the available levels for each leg $\boldsymbol{\ell}$. The mechanism works by making travel time and fuel consumption for each leg function of the chosen speed level.

Constraint (9) ensures the feasibility of cargo capacity. It guarantees that the ship assigned to a route has sufficient capacity to meet the minimum cargo requirement for each leg.

Constraints (10)-(13) model the schedule and delays (schedule & limit time). Constraint (10) initializes the time at the start of the route. Constraint (11) propagates the arrival time between ports sequentially, i.e., the arrival time at the next port must be at least equal to the previous arrival time plus the service time and the travel time at the selected speed. Then, constraint (12) defines the lateness variable per leg as the difference between actual duration and the leg's deadline. At the same time, constraint (13) ensures that lateness is non-negative, so penalties are incurred only when delays occur.

Constraints (14)-(17) model fuel inventory balance and tank capacity limits. Constraint (14) sets the initial onboard fuel inventory of ship $\boldsymbol{k}$ on route $\boldsymbol{r}$ at the first port of the route. The initial inventory is defined as a fraction $\boldsymbol{\beta}$ of the vessel's fuel tank capacity and is activated only when ship $\boldsymbol{k}$ is assigned to route $\boldsymbol{r}$. Constraint (15) states the relationship between fuel inventory upon departure and the decision to purchase bunkering at the port. Constraint (16) models the fuel balance along the leg: the inventory upon arrival at the next port equals the inventory upon departure minus the fuel consumption for that leg, which depends on the selected speed. Constraint (17) ensures that bunkering decisions can only be made at ports with bunkering facilities and keeps the decision and inventory within tank capacity limits (active only when a ship is assigned).

Constraint (18) defines the segment consumption $\boldsymbol{SegCons_{k,r,p}}$ as the total fuel required to travel from the current bunkering port $\boldsymbol{p}$ to the next bunkering port $\boldsymbol{nb(r,p)}$. This quantity is computed by summing the speed-dependent fuel consumption over all legs within that segment, weighted by the speed-selection variable $\boldsymbol{y_{k,r,l,s}}$. The constraint is imposed for ports with $\boldsymbol{A_{r,p} = 1}$, since the segment is anchored at a bunkering decision point.

Constraint (19) defines $\boldsymbol{LegCons_{k,r,p}}$ as the fuel consumption of ship $\boldsymbol{k}$ on route $\boldsymbol{r}$ for the immediate next leg departing from port $\boldsymbol{p}$, namely the leg from port $\boldsymbol{p}$ to port $\boldsymbol{p+1}$. The selected speed level determines its value through $\boldsymbol{y_{k,r,p,s}}$.

Constraint (20) defines the adaptive safety stock requirement. For an immediately reachable bunkering port, the safety stock equals $\boldsymbol{\alpha}$ times the next-leg fuel consumption. For a farther bunkering port, it equals the remaining segment requirement beyond the next leg plus its safety factor.
Constraint (21) enforces the adaptive minimum arrival fuel requirement. Specifically, the arrival fuel inventory $\boldsymbol{I^{arr}_{k,r,p}}$ must be at least as large as the required adaptive safety stock $\boldsymbol{ReqSS_{k,r,p}}$ defined in Constraint (20), thereby ensuring that the vessel retains a sufficient fuel reserve based on the distance to the next feasible bunkering opportunity.

Constraints (22)–(33) define the feasible bounds of onboard fuel inventory and link them to the vessel assignment decision. Specifically, Constraint (22) enforces that the arrival fuel inventory $\boldsymbol{I^{arr}_{k,r,p}}$ is non-negative and cannot exceed the vessel's fuel tank capacity $\boldsymbol{Cap^{tank}_{k}}$; this upper bound is activated only when vessel $\boldsymbol{k}$ is assigned to route $\boldsymbol{r}$ through the binary variable $\boldsymbol{x_{k,r}}$. Similarly, Constraint (23) imposes the same non-negativity and tank-capacity upper bound on the departure fuel inventory $\boldsymbol{I^{dep}_{k,r,p}}$, ensuring that inventory levels remain physically feasible and become zero (inactive) when the vessel is not deployed on the corresponding route.

Parameters

This section explains how raw case data are converted into model-ready parameters. The parameters are grouped into: (i) derived parameters computed from vessel/route inputs, (ii) leg time-limit (deadline) estimation used to evaluate lateness penalties, and (iii) linearization of sailing time specifically introduced to keep the holding-cost component linear in the MILP.

Derived parameters

Several key parameters are not directly available in the operational dataset and are therefore derived before optimization.

The maximum fuel tank capacity for each vessel is estimated using a linear regression relationship that links the main fuel tank capacity to the vessel's deadweight tonnage (DWT), as expressed in Eq. (24). The resulting capacity is then converted into mass units (MT) using the assumed fuel density, as shown in Eq.(25) [22].

$$\boldsymbol{Cap^{tank}_{k} = DWT \cdot \beta^{DWT} + Intercept^{DWT}} \tag{24}$$

$$\boldsymbol{Cap^{tank}_{k} = \rho \cdot Cap^{tank}_{k}} \tag{25}$$

For each route and leg, the sailing distance (NM) is converted to sailing time (hours) for each vessel and discrete speed level using Eq. (26).

$$\boldsymbol{t_{k,r,\ell,s} = \frac{d_{r,\ell}}{v_{k,s}}} \tag{26}$$

The fuel consumption required to traverse a leg is computed using Eq. (27), which combines the selected speed level and the corresponding sailing time.

$$\boldsymbol{cons_{k,r,\ell,s} = FR_{k,s} \cdot t_{k,r,\ell,s}} \tag{27}$$

The holding-cost rate is derived from the annual inventory carrying cost rate and the relevant fuel value basis Eq. (28), then converted to a compatible unit (e.g., cost per MT per hour) for the planning horizon [30].

$$\boldsymbol{h = \frac{carrying\ rate\ per\ year\ \times \overline{price}}{8760}} \tag{28}$$

Estimated time limit per leg

The limit time per leg is determined using the following: First, the planned speed (knots) is determined as the company's standard speed policy, i.e., it is a fixed value based on company policy. In this study, the leg time limit is constructed using two components:

Planned Travel Time: The planned travel time for each leg is computed using Eq. (29), based on the leg distance and a planned/reference speed.

$$\boldsymbol{t_{r,\ell}^{plan}} = \frac{\boldsymbol{d_{r,\ell}}}{\boldsymbol{v^{plan}}} \tag{29}$$

Buffer Time: A buffer is added to account for operational uncertainty and ensure schedule robustness. The buffer is defined as a percentage of the planned time, subject to minimum and maximum bounds, as formulated in Eq. (7)[31].

$$\Delta_{\boldsymbol{r,\ell}} = \boldsymbol{t_{r,\ell}^{plan}} + \mathbf{min}\left(\boldsymbol{B^{max}}, \mathbf{max}\left(\boldsymbol{B^{min}}, \boldsymbol{\gamma} \cdot \boldsymbol{t_{r,\ell}^{plan}}\right)\right) \tag{30}$$

Linearization for Holding-cost Computation

In the integrated model, sailing time depends on discrete speed choices. If holding cost is computed using decision-dependent sailing time together with inventory variables, the cost term can become nonlinear. To preserve a MILP structure, this study linearizes the holding-cost calculation by using a fixed sailing time. Specifically, Eq. (6) is used not only to compute the planned travel time for constructing leg time limits, but also to define a linearized sailing time for the holding-cost component. With this approach, holding costs remain linear in the inventory variables, while the speed-dependent sailing time is preserved where it is operationally critical.

Safety Stock Policy

The static safety stock baseline is set to 10% of fuel tank capacity, while the proposed adaptive policy applies a 20% safety factor to the estimated fuel consumption per sailing segment. This parameterization is intentionally conservative to ensure that any observed cost reduction is attributable to the adaptive mechanism's structural flexibility rather than to a relaxation of fuel security standards. The choice of a 20% factor aligns with service-level-based safety stock formulations in inventory planning, where safety stock is introduced as a buffer against uncertainty and is commonly parameterized by a safety factor (k) associated with the desired service level [25]. Moreover, maritime fuel inventory studies also express safety stock as a percentage of total fuel supplied, reporting route-dependent values (e.g., approximately 15-29%) and noting that operators may apply higher buffers based on field judgment[32].

Adaptive Safety Stock Allocation at each Port

The key novelty of this study lies in introducing an Adaptive Safety Stock allocation at each port and embedding it directly into the integrated MILP model. Unlike a static safety stock policy that applies a uniform buffer level across ports, the proposed mechanism allocates safety stock based on segment exposure, acknowledging that bunkering is only available at a subset of ports and that the distance and conditions govern the operational fuel risk at a given port to the next feasible bunkering opportunity along the route.

Specifically, the *Adaptive Safety Stock* mechanism is designed to dynamically respond to network topology variability by integrating two key variables: *Leg Consumption* (LegCons), representing fuel consumption to the immediate next port, and *Segment Consumption* (SegCons), representing accumulated consumption up to the next bunkering port. This mechanism operates based on a precise conditional rule: (1) on normal segments where the destination port has bunkering facilities, the safety stock threshold is calibrated proportionally only to (LegCons); whereas (2) on long legs passing through non-bunkering ports, the inventory mandate is extended to cover the total remaining journey (SegCons - LegCons) along with its safety factor. This approach guarantees energy supply at critical points along long routes while minimizing unproductive inventory on shorter legs.

Results and Discussions

Table 3 presents strategic fleet deployment decisions, demonstrating the allocation of heterogeneous vessels based on the alignment of technical characteristics and cost structures with the demand profile of each route. To minimize total costs, the model strategically selects vessels by matching their technical specifications (TEU and fuel capacity) with route-specific cargo demands and sailing distances. The algorithm intelligently balances cost-efficiency with operational feasibility, ensuring the assigned ships can maintain the required speeds for timely arrivals without experiencing fuel shortages. The optimization results reveal a clear segmentation, wherein Routes 1, 2, and 3 are served by small-to-medium-sized vessels (Types 1, 2, and 3) with carrying capacities of 115–125 TEU and efficient operational costs ranging from 70–72 USD/hour, reflecting the moderate cargo volumes on these routes. To illustrate the model's decision-making, consider the assignment for Route 5, which exhibits the highest cargo demand per leg. To accommodate this peak volume, the optimization algorithm exclusively deploys Ship 5. This selection is highly logical as Ship 5 offers the maximum capacity (290 TEU), ensuring the route's high demand is fully met without violating any capacity constraints.

Conversely, Routes 4 and 5 are allocated large-capacity vessels (Types 4 and 5) with cargo spaces of up to 290 TEU and substantial fuel tank capacities of up to 620 MT, which are essential for serving long-haul or high-volume routes. This deployment pattern validates the model's ability to optimize asset utilization by preventing overcapacity on low-demand routes while leveraging economies of scale on high-demand routes to minimize total fixed operational costs.

Table 3. Fleet deployment results

Route	Ship code	Load capacity (TEU)	Fuel tank capacity (MT)	Operational non-fuel cost (USD/hour)
1	1	125	279	72.57
2	2	115	248	70.78
3	3	125	248	71.42
4	4	260	530	90.40
5	5	290	620	93.41

Table 4. Optimization of sailing speed result

leg	Speed (knot)	Dep. time (hour)	Arr. time (hour)	Sailing time (hour)	Limit time (hour)	Lateness (hour)
Route 1						
1	7.3	5	57.4	62.4	60.2	0
2	7.3	67.4	87.6	155	91.9	0
3	7.3	160	63.9	223.9	67	0
4	7.3	228.9	100.9	329.8	105.9	0
Route 2						
1	7.35	5	62.8	67.8	66.4	0
2	7.35	72.8	19.8	92.6	21	0
3	6,4	97.6	5.9	103.5	5.7	0.2
4	7.35	108.5	15.3	123.8	16.2	0
5	7.35	128.8	18.6	147.4	19.7	0
6	7.35	152.4	22.3	174.7	23.6	0
7	7.35	179.7	11	190.7	11.6	0
8	7.35	195.7	33.1	228.8	34.9	0
9	7.35	233.8	62.8	296.6	66.4	0
Route 3						
1	6.9	5	144.7	149.7	143.5	1.2
2	6.9	154.7	3.4	158.1	3.9	0
3	6.9	163.1	146.5	309.6	145.3	1.2
Route 4						
1	7.2	5	177.5	182.5	183.7	0
2	7.2	187.5	13.8	201.3	14.3	0
3	7.2	206.3	18.3	224.6	18.9	0
4	7.2	229.6	12.2	241.8	12.7	0
5	7.2	246.8	162.7	409.5	168.3	0
Route 5						
1	9.2	5	45.5	50.5	60.2	0
2	9.2	55.5	75	130.5	99.2	0
3	9.2	135.5	4.3	139.8	5.9	0
4	9.2	144.8	2.3	147.1	3.7	0
5	9.2	152.1	3.8	155.9	5.4	0
6	9.2	160.9	109.5	270.4	144.7	0

Table 4 summarizes the optimized sailing speed decisions and schedule adherence across all five routes. The results demonstrate a strategic variation in speed profiles to balance fuel efficiency with schedule compliance. After assigning vessels to their respective routes, the model serves as a decision-support system that calculates the most cost-effective sailing speed for each leg of the voyage. To generate these numbers, the algorithm simultaneously evaluates the geographical distance of each leg, the available sailing time window, and the strict deadline (Limit Time). The model mathematically weighs the cost of fuel consumption (which decreases at slower speeds) against the penalty costs for schedule delays. By processing these variables, the solver outputs the exact optimal speed required to achieve the lowest possible total operational cost while maintaining schedule feasibility. For Routes 1, 4, and 5, the model selects a consistent sailing speed for each leg (7.3 knots, 7.2 knots, and 9.2 knots, respectively). These speeds are sufficient to meet service deadlines with zero lateness for most voyages.

In contrast, Routes 2 and 3 exhibit a calculated trade-off between energy efficiency and punctuality. Route 2 adopts a variable-speed profile, reducing speed to 6.4 knots on Leg 3, resulting in a negligible 0.2-hour delay. Similarly, Route 3 operates at a constant low speed of 6.9 knots, leading to minor delays of 1.2 hours on Legs 1 and 3. This behavior indicates that the optimization model prioritizes slow steaming strategies to minimize fuel consumption, the dominant operational cost. The minor penalty costs incurred from these slight delays are outweighed by the significant fuel savings, demonstrating an economically efficient operational balance.

Table 5. Optimization bunkering strategy result

Port	I^{arr} (MT)	LegCons (MT)	SegCons (MT)	Req SS (MT)	Min. target inventory (MT)	Min. target bunkering (MT)	Bunkering total (MT)	I^{dep} (MT)
Route 1								
1	3	18	18	4	21	19	104	106
2	88	27	27	5	33	0	0	88
3	61	20	51	38	57	0	0	61
4	42	31	31	6	38	0	0	42
5	10	0	0	0	0	0	0	10
Route 2								
1	2	27	27	5	32	30	30	32
2	5	8	17	10	18	13	53	59
3	50	2	8	8	10	0	0	50
4	49	7	7	1	8	0	0	49
5	42	8	36	34	42	0	0	42
6	34	10	28	23	32	0	0	34
7	24	5	19	17	22	0	0	24
8	20	14	14	3	17	0	0	20
9	6	27	27	5	32	27	27	32
10	5	0	0	0	0	0	0	5
Route 3								
1	2	45	91	56	101	98	98	101
2	56	1	46	55	56	0	0	56
3	55	45	45	9	55	0	0	55
4	9	0	0	0	0	0	0	9
Route 4								
1	5	91	91	18	109	103	207	213
2	122	7	7	1	8	0	0	122
3	115	9	9	2	11	0	0	115
4	106	6	6	1	8	0	0	106
5	100	83	83	17	100	0	0	100
6	17	0	0	0	0	0	0	17
Route 5								
1	6	33	33	7	39	33	183	189
2	156	54	139	103	156	0	0	156
3	103	3	86	99	102	0	0	103
4	100	2	83	97	99	0	0	100
5	98	3	81	94	97	0	0	98
6	95	78	78	16	94	0	0	95
7	17	0	0	0	0	0	0	17

Table 5 summarizes the optimal bunkering strategies shaped by fuel price differentials and the adaptive safety stock mechanism. The bunkering optimization model determines procurement locations and volumes by balancing fuel prices against inventory holding costs. It calculates operational requirements using Leg Consumption (LegCons) for the immediate next port and Segment Consumption (SegCons) to reach the next bunkering facility. By establishing a target inventory based on SegCons plus a 20% adaptive safety stock, the model guarantees operational feasibility. Consequently, it strategically bunkers at cheaper ports, allowing the vessel to safely bypass expensive stops while maintaining sufficient reserves.

This implementation is evident in the dominance of refueling at the initial ports for Route 1, which offers competitive pricing to satisfy the requirements of long-haul segments. Conversely, Route 2 exhibits flexibility through partial refueling at multiple ports to cover dynamic target gaps across shorter voyage segments.

Table 6 presents the total operating costs for the five routes analyzed, which aggregate fleet costs, bunkering costs, fuel-holding costs, and lateness penalties. The calculations show that the total costs for each route are USD 156,934 (Route 1), USD 167,294 (Route 2), USD 157,955 (Route 3), USD 314,655 (Route 4), and USD 267,992 (Route 5), resulting in a cumulative network cost of USD 1,064,830.

Table 6. Total cost summary

Route	Operational cost (USD)	Bunkering cost (USD)	Holding cost (USD)	Penalty cost (USD)	Total (USD)
1	23.940	132.180	814	-	156.934
2	21.021	145.586	313	374	167.294
3	22.117	130.366	740	4.732	157.955
4	37.033	275.518	2.104	-	314.655
5	25.269	242.572	151	-	267.992
Total					1.064.830

Cost disparities between routes are primarily driven by variations in fuel consumption, route length, and the bunkering patterns generated by the model. Overall, these results demonstrate that the optimization model can determine efficient operational strategies for each route by minimizing total network costs while strictly adhering to inventory, speed, and scheduling constraints.

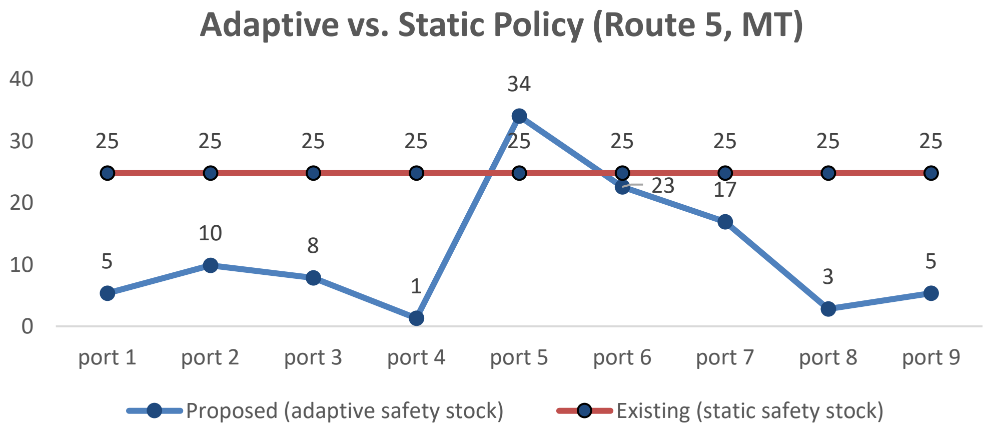

Figure 2. Safety stock allocation across ports: Adaptive vs. static policy (Route 5, MT)

To validate the methodological contribution of this research, as illustrated in Figure 2, a comparative analysis was conducted between the Existing Scenario, which implements a conventional Static Safety Stock policy, and the Proposed Scenario, which utilizes an Adaptive Safety Stock mechanism. In this comparison, the Proposed Scenario was intentionally subjected to a stricter safety constraint (20%) compared to the Existing Scenario (10%). This parameter design aims to objectively demonstrate that the resulting cost efficiency is not due to a reduction in energy supply security standards but rather stems purely from the model's structural flexibility in responding to dynamic operational needs. This fundamental difference is illustrated in Figure 2 (Route 5), where the static line (blue) appears rigid and flat because it is based on a fixed tank capacity, in contrast to the adaptive line (green), which fluctuates responsively in response to the estimated fuel consumption of the subsequent segment.

The comparative analysis of bunkering volumes, as illustrated in Figure 3, highlights the significant efficiency gains achieved through the proposed adaptive safety stock mechanism. The data reveal a marked reduction in total fuel procurement in the proposed scenario compared to the existing static approach. Specifically, substantial volume decreases are observed at major bunkering nodes, such as Port 5 (from 228 MT to 182.66 MT) and Port 4 (from 244 MT to 207.47 MT). This overall reduction validates that the adaptive policy effectively eliminates unnecessary fuel inventory by calibrating safety stocks to actual leg-specific consumption needs rather than relying on rigid, static buffers. Consequently, the model enables leaner operations without compromising supply security.

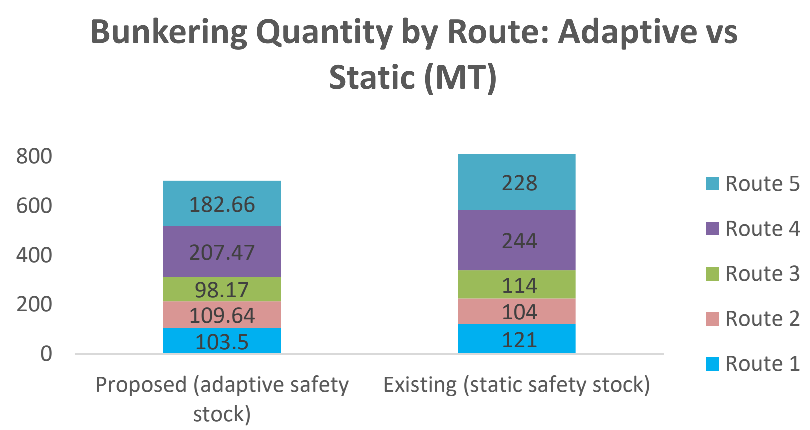

Figure 3. Bunkering Quantity by Route: Adaptive vs Static (MT)

Table 7. Comparison of financial impact

Component Cost	Proposed Model	Existing Model
Total operational cost (USD)	129.380	131.965
Total bunkering cost (USD)	926.222	1.075.585
Total holding cost (USD)	4.122	7.517
Total penalty cost (USD)	5.106	50.318
Total cost (USD)	1.064.830	1.265.385

The table above summarizes the financial impact of implementing the two inventory policy scenarios. Simulation results indicate that the Adaptive Safety Stock method generates substantial cost savings, with a total network cost of USD 1,064,830—approximately 18.89% more efficient than the Static Safety Stock scenario, which reached USD 1,265,385. The most significant efficiencies stem from the reduction in Bunkering Costs (from USD 1.075 million to USD 926,000) and Holding Costs (a decrease of approximately 45%). These findings confirm that eliminating excess inventory directly alleviates unnecessary fuel procurement burdens and capital holding costs. Interestingly, the Total Operational Cost components remain identical or nearly identical in both scenarios, indicating that these significant savings were achieved without compromising service quality (vessel speed) or schedule reliability (lateness). Conversely, a significant disparity exists in the penalty costs between the two scenarios, as the existing scenario utilizes minimum speed levels.

Sensitivity Analysis

To evaluate the robustness and adaptability of the proposed optimization model under market uncertainties, a comprehensive sensitivity analysis was conducted. Table 8 presents the results of this analysis, illustrating how fluctuations in fuel prices affect key operational decisions, such as fleet deployment, speed profiles, and bunkering volumes. The results show that these operational decisions remain stable across all scenario fuel prices. This stability indicates that the baseline scenario already operates at maximum efficiency under slow-steaming policies, meaning vessels cannot further reduce speeds without violating strict service time windows. Interestingly, a structural behavioral shift occurs only under extreme market conditions, specifically at a +50% fuel price surge, when the model rationally reduces the bunkering volume on Route 2 from 110 to 83 units. This specific adjustment reveals the intelligent financial fail-safe built into the adaptive safety stock mechanism; when inventory holding costs become prohibitively expensive, the model dynamically transitions to a conservative Just-In-Time (JIT) procurement strategy to protect cash flow, validating the model's capability to balance operational reliability during normal conditions with financial risk mitigation during extreme price shocks.

Table 8. Sensitivity Analysis

Decision	Route	Fuel price scenarios						
		-50%	-20%	-10%	Baseline	+10%	+20%	+50%
Fleet deployment (Ship Code)	1	1	1	1	1	1	1	1
	2	2	2	2	2	2	2	2
	3	3	3	3	3	3	3	3
	4	4	4	4	4	4	4	4
	5	5	5	5	5	5	5	5
Sailing speed (Speed Level)	1	1	1	1	1	1	1	1
	2	2	2	2	2	2	2	2
	3	1	1	1	1	1	1	1
	4	1	1	1	1	1	1	1
	5	1	1	1	1	1	1	1
Bunkering (MT)	1	100	100	100	100	100	100	100
	2	110	110	110	110	110	110	83
	3	98	98	98	98	98	98	98
	4	207	207	207	207	207	207	207
	5	183	183	183	183	183	183	183
Total cost ($)	All	603,003	880,944	973,566	1,064,830	1,158,810	1,251,231	1,523,885
% Total cost changes	All	-43%	-17%	-9%	0%	9%	18%	43%

Managerial Implications

Based on the optimization results, this study offers two main managerial implications. First, liner shipping operators should avoid managing fleet deployment, sailing speed, bunkering decisions, and fuel reserve policies in isolation. Since these decisions are operationally interdependent, they should be evaluated within an integrated planning framework to improve cost efficiency while maintaining service reliability. In practical terms, this suggests the need for decision-support tools that jointly determine vessel assignment, speed selection, refueling quantities, and minimum fuel reserves across voyage legs.

Second, the findings highlight the managerial relevance of adaptive safety stock for real-world Sea Tollway operations. Rather than applying a uniform fuel reserve policy across all route segments, operators should adjust the minimum onboard fuel inventory based on leg-specific fuel consumption and bunkering accessibility. This enables more efficient fuel planning by reducing unnecessary bunkering and holding costs without compromising service resilience, particularly in geographically dispersed archipelagic networks such as the Indonesian Sea Tollway.

Conclusions

This research successfully developed an integrated optimization model based on Mixed-Integer Linear Programming (MILP) for liner shipping operations within the Indonesian Sea Tollway network. The model simultaneously integrates four key strategic decisions: heterogeneous fleet deployment, sailing-speed optimization, bunkering strategy, and an adaptive safety-stock mechanism. The primary objective of this framework is to minimize total operational costs while strictly adhering to cargo capacity, fuel tank constraints, and service time windows for each sailing leg.

The model's effectiveness was validated through a comparative analysis between the Existing Scenario (utilizing conventional Static Safety Stock parameters) and the Proposed Scenario (employing the Adaptive Safety Stock mechanism). Computational results demonstrate that the Proposed Scenario yields significantly more efficient cost performance compared to the Existing Scenario. Quantitatively, the Proposed Scenario successfully reduced the Total Cost by 18,89%, equivalent to savings of USD 191.555 per service cycle. These savings confirm that integrating these decisions into a single optimization framework provides a more economical solution without compromising operational reliability.

In-depth analysis reveals that the inefficiencies in the Existing Scenario stem from reliance on fixed (static) tank capacity parameters, leading to dead stock accumulation and fuel over purchasing. Conversely, the Proposed Scenario, utilizing the Adaptive Safety Stock mechanism, proved capable of precisely rationalizing inventory levels in response to actual demand fluctuations on segments between bunkering ports. This approach controls fuel shortage risks (stockouts) without burdening vessels with permanently high inventory levels, thereby significantly reducing holding costs.

Beyond cost aspects, the optimization results indicate that fleet deployment decisions determine the system's feasibility in meeting distribution demand. At the same time, sailing speed and bunkering strategies play a dominant role in shaping the total cost structure. The model consistently recommends slow-steaming policies on most legs to minimize fuel consumption, resulting in minimal lateness within acceptable operational tolerances. Practically, it provides a robust decision-support framework for maritime stakeholders to eliminate siloed operations and shift from static to adaptive inventory management, effectively balancing fuel price arbitrage with supply chain resilience.

This study contributes methodologically by offering a comprehensive decision-making framework that supersedes the partial approaches commonly found in literature. As a direction for future research, this model can be extended by incorporating stochastic factors for demand and port service times, and by integrating carbon emissions evaluations to support sustainable green shipping policies.

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