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Modeling Fresh Product Delivery Routes with Heterogeneous Vehicle Routing Problem with Time Windows and Multi-Trips Model: A Case Study

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Abstract: This study develops a Heterogeneous Vehicle Routing Problem with Time Windows and Multi-Trips (HVRPTWMT) model designed to minimize delivery distances for fresh products. The model addresses complex operational constraints inherent in real-world logistics, including time windows, heterogeneous fleets, and multi-trip requirements. A quantitative approach was employed to formulate the HVRPTWMT model, which was then solved using an analytical method to ensure a global optimum solution was found. The model's efficacy was demonstrated through its application to historical data from April 10, 2025, yielding an optimal total distance of 774.45 km across six efficient routes. Sensitivity analysis confirmed the model's robustness and responsiveness to critical parameter changes, such as vehicle capacity, demand fluctuations, and time limits. The developed HVRPTWMT model provides a globally optimal and rule-compliant solution for complex fresh product delivery logistics.

Keywords: Fresh chicken meat, HVRPTWMT, logistics, route optimization, vehicle routing problem.

Introduction

The increasing global population is leading to a rising demand for transportation and logistics, necessitating exceptional efficiency for sustainable economic growth. Efficient movement of goods is crucial for business success, as distribution can represent 10-15% of the selling price [1]. If this process is hindered, it can result in customer dissatisfaction, harm the company's reputation, reduce performance, and even lead to product returns, which is a challenge currently being faced by a company.

As a chicken meat producer, the company reported a 3% return in April 2025, achieving 5.9 tons out of a total of 197 tons due to delays. Demand for chicken meat is anticipated to increase significantly from 2022 to 2026 [2]. Currently, determining the company's delivery routes is a manual and complex process, influenced by several internal constraints. These include tight deadlines for receiving goods at customer delivery points, the need to separate fleets for fresh and frozen products, varying vehicle capacities, limited vehicle availability, and restricted trip capacities.

The complexity of a company's delivery routes can be effectively modeled as the Heterogeneous Vehicle Routing Problem with Time Windows and Multi-Trips (HVRPTWMT). This mathematical modeling approach was selected because it comprehensively addresses the constraints of a heterogeneous fleet, time windows, and multi-trip requirements, ultimately helping to identify optimal and compliant routing solutions. Research in the HVRPTWMT variant has consistently demonstrated its effectiveness in minimizing distances, costs, and delivery times [3, 4]. The study of Vehicle Routing Problems (VRP) has evolved rapidly, utilizing a variety of methods to tackle its inherent complexity [5, 6, 7]. Analytical techniques, such as Arc-flow and GNU Linear Programming Kit (GLPK), aim to find optimal solutions but often require high computing power. For large-scale problems, metaheuristic approaches such as Hybrid Heuristic Harmony Search Algorithm (HHHSA), Adaptive Large Neighborhood Search (ALNS), which is sometimes combined with Variable Neighborhood Descend (VND), and Taboo Search Algorithms are effective choices because they are efficient in finding near-optimal solutions [7, 8, 9]. In addition, hybrid methods such as integrating Constraint Programming (CP), Genetic Algorithm (GA), Mathematical Programming (MP), or combining Mixed-Integer Programming (MIP) and two-stage heuristics combine the strengths of various approaches [10, 11].

This study identifies a gap in prior research concerning the prioritization of Trip 1 deliveries, particularly regarding the use of eight more comprehensive types of constraints within a single model. The objective of this study is to develop a mathematical model of the Home Delivery Vehicle Routing Problem with Multiple Time Windows (HVRPTWMT) to minimize mileage for a company's operations. The model will consider the diversity of vehicle types and capacities, the ability to make multiple trips, and time constraints related to returning to the depot, while also integrating specific constraints to prioritize morning deliveries. Additionally, this study will analyze how sensitive the model is to changes in various parameters, including the number and capacity of vehicles, the quantity and average of customer requests, delivery deadlines, and time windows. The resulting minimum mileage from the model will be compared to the company's existing delivery schedule. The model will be developed using analytical methods with Lingo software serving as the solver.

This paper is structured into several sections to give a comprehensive overview of the research conducted. Part 1 serves as the introduction, while Part 2 outlines the methods employed in the study. Part 3 presents the results and discussion, which includes the developed model, sensitivity analysis, and a comparison of the model with the actual conditions of a company. Finally, Part 4 summarizes the conclusions drawn from this study.

Literature Review

George Dantzig and John Ramser formally started the Vehicle Routing Problem (VRP) in 1959 in their classic paper The Truck Dispatching Problem. Since then, VRP has become a central topic in optimization and logistics research, fueling rapid developments in variants and solution approaches. VRP modeling approaches are generally classified into four main groups: analytical, metaheuristic, hybrid, and machine learning.

In a metaheuristic approach, studies such as Granular Taboo Search (GTS) and a combination of Adaptive Large Neighborhood Search (ALNS) and Variable Neighborhood Descent (VND) are employed. Although the ALNS-VND algorithm is not specifically designed for the MT-VRPTW, the experiment confirms its correctness and competitiveness in solving the MT-VRPTW [8]. GTS has already analyzed thermodynamic models for a VRP problem, which could generate new solutions that focus on reducing the distance-related costs of routes [12]. The hybrid approach combines metaheuristics with predictive, exact, or machine learning methods, e.g., HHHSA, Hybrid Genetic Algorithm (HGA), Hybrid Genetic Algorithm-Solomon Insertion Heuristic (HGA-SIH), Simulated Annealing (SA) with Mixed-Integer Linear Programming (MILP), as well as a combination of Hybrid Genetic Algorithm-Variable Neighborhood Search (HGA-AVNS) and Graph Convolutional Network (GCN), Guided Nondominated Sorting Genetic Algorithm II (G-NSGA-II), and Variable Neighborhood Search-Nondominated Sorting Genetic Algorithm II (VNS-NSGA-II) with Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [7, 10, 14, 15, 16, 17, 18].

HHHSA had a clear advantage in generating high-quality solutions and outperformed many traditional methods in terms of computational efficiency [7]. The use of the HGA approach for each variant of the problem under consideration (with time windows, without time windows, with logical constraints, etc.) significantly reduces the optimization time by up to 100 times compared to the traditional method of implementation — mathematical programming [10]. HGA-SIH demonstrates highly promising results, consistently outperforming state-of-the-art algorithms presented in the existing literature across a range of problem instances [14]. SA can obtain optimal solutions for all small and medium instances with significantly lower computational time [15]. In the case of a combination of HGA-AVNS and GCN, this method reduces carbon emissions by an average of 10.15% compared to ignoring time-dependent speeds and speed fluctuations, and by an average of 23.39% in comparison to the other algorithms [16]. Using G-NSGA-II can improve the CV-GVRP, which could improve customer satisfaction at a lower cost [17]. Finally, real-world case results show that the TDSDGVRPMTW solution proposed in this paper is better than the TDGVRPTW and TDMCGVRPTW solutions when combining VNS-NSGA-II with TOPSIS [18]. Reinforcement learning approaches such as 2D-Ptr are also used for heterogeneous vehicles [19]. Modern approaches based on Reinforcement Learning (RL), such as the RL multi-agent model for real-time VRPTW, demonstrate relevance for intelligent logistics systems [20]. In addition to the above methods, there is also an Evolutionary Algorithm (EA) that provides fuel cost savings [21]. According to the minimum travel distance result of three approximation methods, on average, ALNS had a more efficient 30.02% than SA and 57.21% than GA [22].

Finally, analytical approaches include Branch-and-Price-and-Cut (BPC), the application of GNU Linear Programming Kit (GLPK), and MILP for various scenarios such as fuel distribution, electric vehicles, as well as heterogeneous vehicles and simultaneous services [3, 4, 5, 6, 23]. In GLPK, there are differences between the optimal routes obtained and the existing routes in the company. Despite the difference in the order of cities, the optimal methodology requires the use of 3 vehicles, as opposed to the five vehicles used in the authentic scenario. A reduction in vehicle utilization implies a diminution in total mileage, thereby inferring that the expenses deduced from the model are inferior to tangible expenditures [3]. MILP is used to build mathematical models on VRP, multiple-trip, and multiple product split-delivery problems using the exact branch and bound method to minimize vehicle mileage [4]. Experiments demonstrate that the proposed BPC achieves high efficiency, finding optimal solutions for 82 out of 108 benchmark instances generated by the renowned Solomon's instances, and can solve more instances to optimality in a shorter time [5, 6, 23].

In terms of destination functions, dominance is seen in minimizing total mileage [4]. A wide range of constraint functions guarantees the realism of VRP solutions. Basic constraints include vehicle capacity and number of vehicles [7, 14]. Also consists of the challenges of heterogeneous vehicles [6, 20]. Customer or trip priorities are also starting to be modeled [17]. Also, do not forget the limitations of the unloading station's capacity [5]. Electric vehicles also have limitations on battery capacity and charging [24]. Time and duration constraints are significant, including service time window, maximum route duration, adaptive service time, progressive time window adjustment, as well as unloading time and unloading queues [4, 13]. In the service structure, multi-trip and split delivery concepts are accommodated, along with simultaneous delivery and pickup [8, 10, 23]. Subtour elimination constraints and customer visit sequences are also included [14, 19]. Dynamic models present real-time constraints, such as request insertion and order assignment [13, 20]. Environmental and energy aspects include constraints on carbon emissions, slope, speed, and acceleration, degradation of fresh product quality, as well as cooling systems and fuel consumption [12, 16]. Finally, the constraints of multiskilled labor, service queues at depots, and restrictions on electric vehicle charging stations are also considered [23, 25].

Methods

This study adopts a quantitative approach with a simulation-based experimental design to develop and analyze the optimization model of the Heterogeneous Vehicle Routing Problem with Time Windows and Multi-Trips (HVRPTWMT). The quantitative approach was chosen because the research goal is to minimize mileage, which requires the analysis of measured data and the use of precise mathematical models. Simulation-based experimental design is applied to build and test optimization models in a controlled environment, which systematically evaluates the model's performance in finding optimal route solutions and analyzing its sensitivity to various key parameters.

The developed HVRPTWMT model will be solved using analytical methods with the help of the LINGO 21.0.26 application. This analytical method is crucial because it guarantees optimal solutions (global optimum), which aligns with the research goal of accurately minimizing mileage. The data used as the input model is historical data on the delivery of fresh products from a company on April 10, 2025, which was chosen because it has the highest delivery volume and is relevant to the limitations of the research problem. Using spreadsheet applications will also help in processing raw data from observations, interviews, and literature studies. In contrast, Google Maps is used to determine the geographic coordinate points of customers.

The process began with the identification of specific delivery route problems at a company, which was conducted through interviews and observations, identifying relevant VRP types, and performing a comprehensive literature search to build a theoretical foundation. Once the required data is collected, the core stage of the research proceeds to creating a VRP model in mathematical formulation, which is then implemented into the LINGO application. The model that has been made will undergo an iterative verification and validation process. The model must be error-free (non-infeasible) and capable of generating a logical solution through sensitivity testing before being declared ready to obtain the optimal solution. The result of the model solution will then be compared with the results of the real system of a company, which will evaluate the efficiency and potential improvements that the model can offer, which will ultimately be the basis for formulating conclusions and recommendations.

VRP Model

To formulate an accurate mathematical model, this study uses a series of standard notations that represent various key elements in the route optimization problem.

i, j, k	:	Index for *node* or customer (i is the origin node, j is the destination node, and k is the transit node)
v	:	Index for vehicles
N	:	Total customers
M	:	Total vehicles
H	:	Total Trips
x_{ijv}	:	Decision variable for trips from customer i to customer j on vehicle v (value 1 in case of trip and 0 if otherwise)
d_{ij}	:	Distance from customer i to customer j
q_i	:	Customer's request i
Q_v	:	Vehicle capacity v
u_{iv}	:	Total demand to customers i on vehicle v
v	:	Vehicle speed
Tu	:	Unloading time is 0.08 min/kg
t_{ij}	:	Travel time from customer i to customer j
T_{iv}	:	Arrival time of vehicle v at customer i
Tv_i	:	Time to visit or unload goods to the customer at location i ($q_i.Tu$)
$Tback$	:	Vehicle return time to the depot
a	:	Initial reception time at the customer's location
b	:	Final acceptance time at the customer's location

These notations are the basis for constructing mathematical model equations and constraints to find optimal solutions in determining delivery routes. The VRP model is written as follows:

Objective Function

$$Min \sum_{i}^{N}\sum_{j}^{N}\sum_{v}^{M} d_{ij}\, x_{ijv} \tag{1}$$

Equation (*1*) is the objective function, which mathematically describes the total mileage that must be minimized [7]. The objective function is not a cost function because the route cost of a vehicle is the total of the travel time (proportional to distance), waiting time, and service time required to visit a set of customers [26]. Distance is a key component of the total cost, making it a valid basis for the objective function.

Core VRP Constraints

$$x_{kkv} \leq 0, \forall v, k \tag{2}$$

Equation (*2*) states that vehicles do not travel to the same location on a given route [5], [27].

$$\sum_{i}^{N}\sum_{v}^{M} x_{ikv} = 1, \forall k \tag{3}$$

$$\sum_{j}^{N}\sum_{v}^{M} x_{kjv} = 1, \forall k \tag{4}$$

Equations (*3*) and (*4*) ensure that each customer is visited and departed from by exactly one vehicle [4].

$$\sum_{i}^{N} x_{ikv} - \sum_{j}^{N} x_{kjv} = 0, \forall k, v \in N \tag{5}$$

Equation (*5*) guarantees the continuity of the routing process by ensuring that the number of vehicles arriving at a customer location is equal to the number of vehicles departing or after arriving at the customer, the vehicle will return to the next customer [28].
Subtour Elimination and Capacity Constraints
Equations (*6*) to (*8*) are formulations that combine subtour elimination and capacity management.

$$u_{jv} \geq u_{iv} + q_j x_{ijv} - q_i x_{jiv} + \left(q_j - Q_v\right)\left(1 - x_{ijv} - x_{jiv}\right), \forall v, i, j \tag{6}$$

Equation (*6*) is an extended version of the Miller-Tucker-Zemlin (MTZ) formulation. This equation relates the visit sequence to the vehicle's load. If a vehicle v moves from location i to location j (x_{jiv}=1), this equation ensures that the cumulative load at location j (u_{jv}) is the cumulative load at location i (u_{iv}) plus the demand at location j (q_j). This effectively prevents disconnected routes while simultaneously tracking vehicle loads [27].

$$u_{jv} \leq Q_v - (Q_v - q_k)x_{1jv}, \forall v, j, k \tag{7}$$

$$u_{jv} \geq Q_v + \sum_{i \neq 1}^{N} q_i\, x_{ijv}, \forall v, j \tag{8}$$

Equations (*7*) and (*8*) ensure that the cumulative load never exceeds the maximum vehicle capacity (Q_v), thus guaranteeing a valid route [27].

Decision variable

$$x_{ijv} \in \{0, 1\} \tag{9}$$

Equation (*9*) states that the decision variable x_{ijv} is binary [29].

Time Window Constraints

The time window is an essential constraint in this model because every customer of a company has a specific operating time or receipt time. This means that each vehicle must arrive and serve the customer within a predetermined time frame, ensuring adherence to the receipt schedule.

$$T_{iv} + (Tv_i + t_{ik})x_{ikv} - b_i(1 - x_{ikv}) \leq T_{kv}, \forall v, j, i, k \tag{10}$$

Equation (*10*) calculates the arrival time of a vehicle at a customer location based on its arrival time at the previous location plus travel and service time [27].

$$a_k \leq T_{kv} \leq b_k, \forall v, k \tag{11}$$

Equation (*11*) ensures that a vehicle's arrival time at a customer's location must be within the customer's predefined time window [27].

$$T_{kv} + (Tv_k + t_{k1})x_{k1v} \leq Tback / \left\lceil \frac{\sum_{k \neq 1}^{N} q_k}{\sum_v^M Q_v . N} \right\rceil, \forall v, k \tag{12}$$

Equation (*12*) ensures the vehicle does not exceed the set total trip time limit [28].

Additional Constraints

In addition to the restrictions already mentioned, several additional constraints are implemented to ensure the adequacy of vehicles and prevent trips exceeding the specified trip limits.

$$\left\lceil \frac{\sum_{k \neq 1}^{N} q_k}{\sum_v^M Q_v . N} \right\rceil \leq H \tag{13}$$

Equation (*13*) ensures that the total number of trips formed does not exceed the total number of available trips and makes the first trip take precedence.

$$\sum_{j}^{N} x_{1jv} \leq M . \left\lceil \frac{\sum_{k \neq 1}^{N} q_k}{\sum_v^M Q_v . N} \right\rceil, \forall v \tag{14}$$

Equation (*14*) ensures that the total number of trips formed does not exceed the total number of vehicles and total available trips.

$$\sum_{i \neq j}^{N} \sum_{j \neq i}^{N} q_i x_{ijv} \leq Q_v \left\lceil \frac{\sum_{k \neq 1}^{N} q_j}{\sum_v^M Q_v . N} \right\rceil, \forall v \tag{15}$$

Equation (*15*) provides that the vehicle's capacity is sufficient to fulfill all customer demands.

Geographical and Auxiliary Constraints

$$t_{ij} = v.d_{ij}, \forall j, i \qquad (16)$$

Equations (*16*) define the travel time (t_{ij}) from location i to location j as the distance (d_{ij}) divided by the average vehicle speed (v). This value is crucial for the time window constraints in Equation (*10*). Note: The original formulation ($t_{ij} = v.d_{ij}, \forall j, i$), is a typographical error and has been corrected in the final model to reflect the standard physical relationship.

$$d_{ij} = R.cos^{-1}\left(sin\left(\frac{\pi}{180^o}.lat\ i\right).sin\left(\frac{\pi}{180^o}.lat\ j\right)\right) + cos\left(\frac{\pi}{180^o}.lat\ i\right).cos\left(\frac{\pi}{180^o}.lat\ j\right).cos(|lon\ i - lon\ j|) \qquad (17)$$

Equation (*17*) calculates the distance between two points on the globe. This is the Haversine formula, used to precisely calculate the shortest distance (d_{ij}) between two points on the surface of a sphere (the "Great-Circle" distance), given their latitude and longitude coordinates. This equation grounds the model in real-world geographical data, removing the need for a pre-calculated distance matrix [30].

$$Tv_i = q_i.Tu \qquad (18)$$

Equation *18* calculates the total visit time (Tv_i) required at each location i. It is determined by multiplying the customer's demand (q_i) by a predefined time unit per load, Tu (e.g., minutes per kilogram). This value is then used in Equation (*10*) to accurately compute the total time spent at each customer's location.

Results and Discussions

This section presents the results of the analysis and in-depth discussion related to the optimization research of delivery routes at a company. Starting with an explanation of the existing delivery conditions, it continued with the formulation of the Vehicle Routing Problem (VRP) model used, model validation, and the results of route optimization obtained. Finally, a comparison was made between the results of the model and the real system.

Delivery Conditions

The company's return report from April 2-19, 2025, indicates that shipping delays were responsible for 3% of returns, amounting to 5.9 tons out of 197 tons of total shipments. These delays are primarily caused by the company's reliance on manual route planning, which is a complex process due to strict, rule-based constraints. These operational rules, such as time windows and heterogeneous fleets, are difficult to manage manually but can be precisely formulated as measurable constraints in an optimization model to find a more efficient solution.

A company implements several key shipping policies to ensure efficiency and punctuality. There are two daily delivery times, namely morning and noon, provided all vehicles must return no later than 18.00. Departure times from the factory are flexible and adapted to the given route, and drivers can swap vehicles if needed. It is also important to note the separation of vehicles for fresh (3 L300, 2 CDE) and frozen (6 CDE, 1 CDD) products due to the temperature difference required. In addition, all prepared goods must be delivered according to the schedule, and customer receipt of goods must occur within the agreed time. Finally, morning delivery is prioritized due to the product's short lifespan to ensure that goods reach customers faster.

Model Parameter

The fleet used in this study can be categorized as heterogeneous as it consists of vehicles with different characteristics, namely their carrying capacity (See Table 1). Specifically, this fleet comprises a total of five vehicles divided into two groups: three vehicles with a capacity of 1500 (with license plate numbers S 8932 SB, S 8247 SD, and S 9079 SC) and two vehicles with a capacity of 2500 (with license plate numbers S 9573 SB and S 8298 SD). This diversity in capacity is a crucial factor in VRP modeling, as route and load allocation decisions must be considered the most suitable vehicle for each task.

Table 2 exhibits the model's time and speed parameters. Vehicle speed (TMPM) is 0.83 km/min, unloading time (TMUL) is 0.08 min/kg (based on the interview with the expedition head, the full unloading time for a 2500 kg vehicle is about 2 hours 20 minutes, which is converted to 0.08 minutes/kg), and return to depot time (TBACK) is 1080 minutes. The maximum number of trips (TRIP) is 2. These parameters are a crucial basis in building and optimizing the delivery route model for a company.

Table 1. Vehicles' parameter

Capacity	Number of vehicles	Police number
1500	3	S 8932 SB
		S 8247 SD
		S 9079 SC
2500	2	S 9573 SB
		S 8298 SD

Table 2. Setting parameter

Parameter	Unit	Value
Vehicle Speed (TMPM)	km/min	0,83
Unloading time (TMUL)	min/kg	0,08
Comeback time (TBACK)	min	1080
Trip	-	2

The fleet will serve 10 fresh chicken customers from a depot that acts as the point of origin and return (Table 3). Each customer is identified by a unique Customer ID and has demand data listed in the Total column. The location of each customer is specified by its latitude and longitude. The most critical information for VRP modeling is the time window, which sets the earliest (TME) and latest (TML) arrival times at each customer's location, ensuring deliveries are completed on schedule.

Table 3. Fresh chicken customers' parameters

Customer ID	Total	Latitude	Longitude	TME	TML
DEPOT	0.00	-7.41687100	112.40060400	0	9,999
CEBA000806	101.40	-7.25787250	112.64610330	420	1,020
CEBA000736	113.80	-7.19538050	111.94815300	300	900
CEBA000568	118.40	-7.30256520	112.70383820	300	960
CEBA001517	484.80	-7.94073320	112.62360250	360	960
CEBA000663	513.40	-7.33279250	112.71798040	300	1,020
CEBA000380	514.00	-7.48919720	112.44805300	360	1,020
CEBA000108	571.50	-8.12929270	112.21536400	420	1,020
CEBA001468	695.00	-7.87730030	112.68805390	300	960
CEBA000180	762.70	-7.54546360	112.22906300	420	900
CEBA001108	1,004.90	-7.71850730	112.99892350	360	900
CEBA000380	1,500.00	-7.48919720	112.44805300	360	1,020
CEBA001468	1,500.00	-7.87730030	112.68805390	300	960
CEBA001468	1,500.00	-7.87730030	112.68805390	300	960
CEBA001468	1,500.00	-7.87730030	112.68805390	300	960

Results

Figure 1 shows visual map that represents the optimized routes operate from a main depot to serve customers in various locations. The orange-colored areas in the image represent customer clusters that are grouped into the same delivery route.

The minimum distance is 774.45 km. which is divided into six delivery routes. These routes utilize two vehicle capacities: 1500 kg and 2500 kg. Route 1 (capacity 1500 kg) starts from the DEPOT to the CEBA000736 (LOADCUM 113.8), then to CEBA000180 (LOADCUM 876.5), before returning to the DEPOT. Route 2 (capacity 1500 kg) is a short route from DEPOT to CEBA000380 (LOADCUM 1500.0) and directly back to DEPOT.

For larger capacity vehicles (2500 kg), Route 3 serves four customers: from the DEPOT to CEBA001108 (LOADCUM 1004.9), continuing to the CEBA000663 (LOADCUM 1518.3), to the CEBA000568 (LOADCUM 1636.7), and ending at the CEBA000806 (LOADCUM 1738.1) before returning to the DEPOT. Route 4 moves from DEPOT to CEBA001468 (LOADCUM 1500.0), then to CEBA000108 (LOADCUM 2071.5), and back to

DEPOT. Route 5 starts from the DEPOT to the CEBA001468 (LOADCUM 1500.0), then to the CEBA001517 (LOADCUM 1984.8), to the CEBA000380 (LOADCUM 2498.8), and back to the DEPOT. Finally, Route 6 (capacity 2500 kg) involves traveling from DEPOT to CEBA001468 (LOADCUM 1500.0) and then back to the CEBA001468 (LOADCUM 2195.0), which may indicate repeated visits or separate deliveries to the exact location, before returning to the DEPOT.

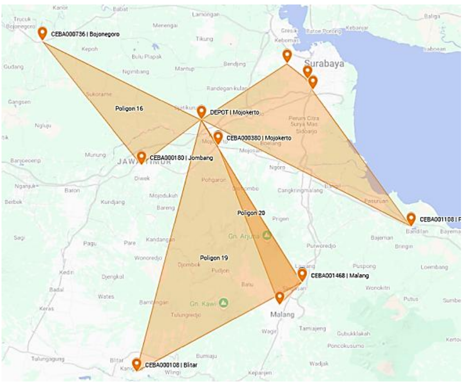

Figure 1. Delivery route maps

Model Comparison with Real Systems

Table 4 shows the delivery distance carried out by a company on April 10, 2025. The total distance traveled by all vehicles is 694.39 km. The figure of 694.39 is lower than the model results due to the improper use of vehicles, specifically 6000 kg vehicles used to transport customer requests CEBA001468, where the demand is 5195, resulting in no split of these customers. Using this inappropriate vehicle does not occur in the model because the model refers to the rules set beforehand. Overall, the model's results are considered superior to the real systems due to its strict adherence to all operational rules. Although the total distance is higher than the real system, the results model can still be considered globally optimal due to the use of vehicles not included in the designated model.

Table 4. Total distance and quantity

Vehicle Police' Number	Departure	Arrival	Distance	Qty
S 8247 SD	DEPOT	CEBA001517	63,22	484,80
	CEBA001517	DEPOT	63,22	0,00
	Total S 8247 SD		126,44	484,80
S 8932 SB	DEPOT	CEBA000568	35,77	118,40
	CEBA000568	CEBA000663	3,71	631,80
	CEBA000663	CEBA000806	11,50	733,20
	CEBA000806	DEPOT	32,34	0,00
	Total S 8932 SB		83,32	733,20
S 9079 SC	DEPOT	CEBA000380	9,59	2.014,00
	CEBA000380	DEPOT	9,59	0,00
	Total S 9079 SC		19,18	2.014,00
S 8298 SD	DEPOT	CEBA000180	23,71	762,70
	CEBA000180	CEBA000736	49,75	876,50
	CEBA000736	DEPOT	55,65	0,00
	Total S 8298 SD		129,11	876,50
S 9573 SB	DEPOT	CEBA000108	81,80	571,50
	CEBA000108	DEPOT	81,80	0
	Total S 9573 SB		163,60	571,50
S 9818 SB	DEPOT	CEBA001108	73,99	1,004,90
	CEBA001108	CEBA001468	38,53	6.199,90
	CEBA001468	DEPOT	60,21	0,00
	Total S 9818 SB		172,73	6.199,90
	Total		694,38	10.879,90

Model Verification and Validation

In this research, a sensitivity analysis was conducted to evaluate the stability and robustness of the optimal solution for the Vehicle Routing Problem (VRP) when key parameters were changed to reflect dynamic real-world conditions. The main objective was to assess the impact of four primary factors: vehicle composition and capacity, customer demand volume, delivery deadlines, and customer time windows. By systematically manipulating these variables, the sensitivity analysis provides a comprehensive understanding of the model's flexibility and efficiency. This approach helps identify the best fleet configurations and operational strategies that are not only optimal for a single scenario but also adaptable to various unexpected changes in logistics.

Table 5. Sensitivity analysis by changing the composition and capacity of the vehicle

Testing	Basis	1	2	3	4
Number of customers	50	50	50	50	10
Average customer demand	100	100	100	100	100
Number of vehicles 1	1	1	1	1	1
Average vehicle capacity 1	10000	5000	2500	3000	1500
Number of vehicles 2				1	1
Average vehicle capacity 2				2000	1000
Number of vehicle routes 1	1	2	3	2	3
Number of vehicle routes 2					1
Total routes	1	2	3	2	4
Total distance	1100	1120	1140	1120	1160

Table 5 illustrates how fleet capacity and composition significantly influence the number of trips, routes, and total mileage. The number of customers and their demand remained unchanged throughout the study, with no restrictions on trips, time windows, or return times. In this sensitivity test, 50 customers were analyzed, each with a demand of 100 kg. In Tests 1 and 2, a reduction in the capacity of the first type of vehicle led to a substantial increase in the number of trips, routes, and total distances. More trips were necessary to transport the same road, which also contributed to increased total distance due to the repeated round-trip calculations from the depot. Conversely, Tests 3 and 4, which involved the second type of vehicle, demonstrated a decrease in total trips and routes. This indicated that enhancing the fleet's overall capacity resulted in improved efficiency. In Test 3, only the first type of vehicle was used, which minimized distance. However, in Test 4, an even distribution of demand between the two types of vehicles proved to be optimal for utilizing fleet capacity. As more routes were formed, the total distance increased due to the necessity of repeated calculations for the distance to and from the depot, as more routes and trips were created.

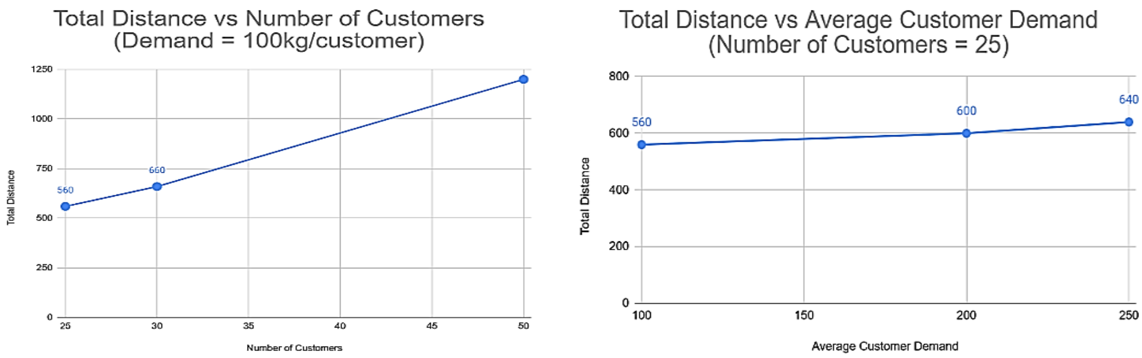

Figure 2. Test model sensitivity by changing the number of requests

Figure 2 presents two graphs that visualize the relationship between the number of customers and the average demand per customer relative to total distance travel, demonstrating that both factors contribute to the increase in distance travel. The graph on the left illustrates that the number of customers has a significant impact, increasing the number of customers from 25 to 50 results in a dramatic increase in the distance traveled, from 560 km to 1200 km. This indicates that serving more customers requires significantly longer routes. On the other hand, the graph on the right shows that the average demand per customer also affects the distance, albeit with a more minor impact; increasing the demand from 100 kg to 250 kg only results in a slight increase in distance from 560 km to 640 km. Overall, these two graphs clearly illustrate that the more customers served and the greater their demand, the longer the distance required. This sensitivity test shows that the model created is quite responsive to changes in demand and the number of customers.

Table 6 shows that changes in delivery deadlines (TBACKs) significantly affect the number of trips and total mileage, although the number of customers and total demand remain constant. When TBACK was capped at

500 in Test 1, operations were unaffected compared to base conditions, with a huge time constraint. This indicates that the limitation was still loose enough not to affect operations with the current fleet configurations. However, the drastic drop in TBACK to 200 in Test 2 significantly increased the number of trips and total distance, highlighting that strict time constraints forced more trips due to suboptimal loadouts, triggering the formation of multiple trips or routes. In contrast, the easing of TBACK to 300 in Test 3 and 400 in Test 4 gradually reduced the number of trips and total distance, returning to close baseline conditions. The logical implication of this test is that overly strict delivery time limits can increase mileage through increased travel numbers and potentially lower vehicle and route utilization efficiency. In contrast, more realistic and flexible time constraints enable better route planning and maximum loading, ultimately contributing to more efficient operations with reduced travel numbers and mileage.

Table 6. Sensitivity analysis by changing the delivery deadline

Testing	Basis	1	2	3	4
Number of customers	50	50	50	50	50
Average customer demand	100	100	100	100	100
Delivery time (TBACK)	9999	2000	200	300	400
Number of vehicle routes 1	2	2	5	4	2
Number of vehicle routes 2					
Total routes	2	2	5	4	2
Total distance	1140	1140	300	280	240

Table 7. Sensitivity analysis by changing the customer's Windows Time

Testing	Basis	1	2	3	4
Number of customers	10	10	10	10	10
Average customer demand	100	100	100	100	100
Number of customers whose time was changed	0	5	4	7	7
TME	0	360	360	360	300
TML	9999	420	420	420	720
Number of vehicle routes 1	2	3	2	4	2
Number of vehicle routes 2					
Total routes	2	3	2	4	2
Total distance	240	260	240	280	240

Table 7 shows that the customer delivery time window (Time Windows), which is reflected in the number of customers with time constraints, and the relaxation of their waiting time ranges, directly affects operational efficiency in terms of the number of trips and total mileage. However, the volume of requests and the number of customers remain stable. Test 1 revealed that the time constraints of TME (initial acceptance) and TML (final acceptance time) for some customers (5 customers changed TME to 360 and TML to 420) began to impact route planning, increasing the number of trips and mileage. Testing 2 was conducted with fewer customers whose Time Windows were changed, resulting in a reduction in the number of trips/routes and mileage. An increase in the number of trips and mileage was again seen in Test 3 as the number of customers with changing time constraints increased. In test 4, the importance of Time Windows' flexibility was highlighted; with a much broader waiting timeframe, the system achieved the same results as the baseline conditions, regarding the number of customers and the distance traveled. The logical implication of this data is that the Time Windows relaxation provides crucial flexibility for the route. Optimization, at the same time, the increasing number of customers with strict time constraints significantly increases planning complexity and potentially increases or decreases mileage.

The sensitivity test results show a responsive and logical model to vehicle quantity/capacity changes, customer demand, delivery deadline, and customer time window. This consistency proves a validated and reliable model for analysis, particularly in determining the optimal number of vehicles.

Conclusions

This research successfully developed a model for the Heterogeneous Vehicle Routing Problem with Time Windows and Multi-Trips (HVRPTWMT). The model aims to provide a globally optimal solution that minimizes delivery mileage for a company. By utilizing the Lingo solver, we were able to determine a minimum distance of 774.45 km across six delivery routes, all while complying with the company's operational constraints. An extensive sensitivity analysis demonstrated that the model is highly responsive to changes in important parameters, such as vehicle capacity, customer demand, and time window constraints. Notably, increases in capacity or the relaxation of time restrictions significantly enhanced route efficiency. This validates the model's logic and shows the robustness of its solutions.

The most significant finding of this study is the comparative analysis between the optimal solution and the company's existing operational data. While the company's system records a lower distance of 694.39 km, our analysis revealed that this seemingly shorter distance was achieved by violating a key delivery rule—specifically, using a large-capacity vehicle without a split load for a particular customer. This indicates that the existing operational system prioritizes efficiency over adherence to rules, resulting in a distance that, although appearing shorter, compromises compliance. Consequently, the solution produced by our model is considered superior because it adheres strictly to all operational constraints, even though it results in a slightly longer distance.

This finding validates our model's capability to identify a globally optimal solution and underscores the considerable potential for a company to enhance efficiency through disciplined adherence to logistics regulations. Our research demonstrates that mathematical optimization can serve as an essential tool for identifying and addressing inefficient or non-compliant operational practices.

References

[1] K. V. Narasimha, E. Kivelevitch, B. Sharma and M. Kumar, "An ant Colony optimization technique for solving the min-max multi-depot vehicle routing problem," *Swarm and Evolutionary Computation*, vol. 13, December 2013, doi: https://doi.org/10.1016/j.swevo.2013.05.005

[2] S. Sirajuddin, G. Bhaswara, and A. Gunawan, "Model sistem dinamis industri ayam pedaging dalam memenuhi kebutuhan daging ayam," *Journal Industrial Servicess*, vol. 8, no. 1, p. 53, Jun. 2022, doi: https://doi.org/10.36055/jiss.v8i1.14184.

[3] U. Dedović and B. A. Gušavac, "Optimal vehicle routing in consumer goods distribution: A GNU linear programming kit-based analysis," *Acadlore Transactions on Applied Mathematics and Statistics*, vol. 1, no. 2, pp. 87–95, Sep. 2023, doi: https://doi.org/10.56578/atams010204.

[4] D. S. Ramadhani, N. A. Masruroh, and J. Waluyo, "Model of vehicle routing problem with split delivery multi trips, multi products and compartments for determining fuel distribution Routes," *ASEAN Journal of System Engineering*, vol. 5, pp. 51–55, Dec. 2021, [Online]. Available: http://journal.ugm.ac.id/index.php/ajse

[5] N. Huang, J. Li, W. Zhu, and H. Qin, "The multi-trip vehicle routing problem with time windows and unloading queue at depot," *Transp Res E Logist Transp Rev*, vol. 152, Aug. 2021, doi: https://doi.org/10.1016/j.tre.2021.102370.

[6] S. Madankumar and C. Rajendran, "A mixed integer linear programming model for the vehicle routing problem with simultaneous delivery and pickup by heterogeneous vehicles, and constrained by time windows," *Sādhanā*, vol. 44, no. 2, p. 39, Feb. 2019, doi: https://doi.org/10.1007/s12046-018-1048-y.

[7] Y. Zhang and J. Li, "A Hybrid Heuristic Harmony Search Algorithm for the Vehicle Routing Problem with Time Windows," *IEEE Access*, vol. 12, pp. 42083–42095, 2024, doi: https://doi.org/10.1109/ACCESS.2024.3378089.

[8] B. Pan, Z. Zhang, and A. Lim, "Multi-trip time-dependent vehicle routing problem with time windows," *Eur J Oper Res*, vol. 291, no. 1, pp. 218–231, May 2021, doi: https://doi.org/10.1016/j.ejor.2020.09.022.

[9] D. Taş, O. Jabali, and T. Van Woensel, "A vehicle routing problem with flexible time windows," *Comput Oper Res*, vol. 52, no. PART A, pp. 39–54, 2014, doi: https://doi.org/10.1016/j.cor.2014.07.005.

[10] P. Sitek, J. Wikarek, K. Rutczyńska-Wdowiak, G. Bocewicz, and Z. Banaszak, "Optimization of capacitated vehicle routing problem with alternative delivery, pick-up and time windows: A modified hybrid approach," *Neurocomputing*, vol. 423, pp. 670–678, 2021, doi: https://doi.org/10.1016/j.neucom.2020.02.126.

[11] C. K. Y. Lin, "A vehicle routing problem with pickup and delivery time windows, and coordination of transportable resources," *Comput Oper Res*, vol. 38, no. 11, pp. 1596–1609, Nov. 2011, doi: https://doi.org/10.1016/j.cor.2011.01.021.

[12] J. W. Escobar, J. L. R. Duque, and R. García-Cáceres, "A granular tabu search for the refrigerated vehicle routing problem with homogeneous fleet," *International Journal of Industrial Engineering Computations*, vol. 13, no. 1, pp. 135–150, Dec. 2022, doi: https://doi.org/10.5267/j.ijiec.2021.6.001.

[13] B. Souleymane, D. Oussama, A. Fatima Ezzahra, and R. Fouad, "A Time Window Assignment Strategy for Dynamic Vehicle Routing Problem," in *Transportation Research Procedia*, Elsevier B.V., 2025, pp. 169–176. doi: https://doi.org/10.1016/j.trpro.2025.03.060.

[14] A. Maroof, B. Ayvaz, and K. Naeem, "Logistics Optimization Using Hybrid Genetic Algorithm (HGA): A Solution to the Vehicle Routing Problem with Time Windows (VRPTW)," *IEEE Access*, vol. 12, pp. 36974–36989, 2024, doi: https://doi.org/10.1109/ACCESS.2024.3373699.

[15] V. F. Yu, P. Jodiawan, S. W. Lin, W. F. Nadira, A. M. S. Asih, and L. N. H. Vinh, "Using Simulated Annealing to Solve the Multi-Depot Waste Collection Vehicle Routing Problem with Time Window and Self-Delivery Option," *Mathematics*, vol. 12, no. 3, Feb. 2024, doi: https://doi.org/10.3390/math12030501.

[16] P. Lou, Z. Zhou, Y. Zeng, and C. Fan, "Vehicle routing problem with time windows and carbon emissions: a case study in logistics distribution," *Environmental Science and Pollution Research*, vol. 31, no. 29, pp. 41600–41620, Jun. 2024, doi: https://doi.org/10.1007/s11356-024-31927-9.

[17] D. Wu, J. Li, J. Cui, and D. Hu, "Research on the Time-Dependent Vehicle Routing Problem for Fresh Agricultural Products Based on Customer Value," *Agriculture (Switzerland)*, vol. 13, no. 3, Mar. 2023, doi: https://doi.org/10.3390/agriculture13030681.

[18] D. Wu and C. Wu, "Research on the Time-Dependent Split Delivery Green Vehicle Routing Problem for Fresh Agricultural Products with Multiple Time Windows," *Agriculture (Switzerland)*, vol. 12, no. 6, Jun. 2022, doi: https://doi.org/10.3390/agriculture12060793.

[19] Q. Liu *et al.*, "2D-Ptr: 2D Array Pointer Network for Solving the Heterogeneous Capacitated Vehicle Routing Problem Attention Mechanism; Combinatorial Optimization; Heterogeneous CVRP; Pointer Network; Reinforcement Learning ACM Reference Format," May 2024, doi: https://doi.org/10.5555/3635637.3662981.

[20] Z. Zong, X. Tong, M. Zheng, and Y. Li, "Reinforcement Learning for Solving Multiple Vehicle Routing Problem with Time Window," *ACM Trans Intell Syst Technol*, vol. 15, no. 2, Mar. 2024, doi: https://doi.org/10.1145/3625232.

[21] E. Hendrawan and I. G. A. Widyadana, "Optimasi rute pengiriman dengan heterogeneous fleet vehicle routing problem with time windows," *Jurnal Sistem dan Manajemen Industri*, vol. 2, no. 1, p. 1, May 2018, doi: https://doi.org/10.30656/jsmi.v2i1.518.

[22] R. Fauzi, A. Priansyah, P. K. Puspadewa, S. M. Z. Awal, H. T. Nguyen, and A. P. Rifai, "Optimizing vehicle routing for perishable products with time window constraints," *Jurnal Teknik Industri*, vol. 27, no. 1, pp. 1–20, Jan. 2025, doi: https://doi.org/10.9744/jti.27.1.1-20.

[23] N. Huang, H. Qin, G. Xu, and F. Wan, "An enhanced exact algorithm for the multi-trip vehicle routing problem with time windows and capacitated unloading station," 2024. doi: https://doi.org/10.1016/j.cor.2024.106688.

[24] E. N. Duman, D. Taş, and B. Çatay, "Branch-and-price-and-cut methods for the electric vehicle routing problem with time windows," *Int J Prod Res*, vol. 60, no. 17, pp. 5332–5353, 2022, doi: https://doi.org/10.1080/00207543.2021.1955995.

[25] N. A. El-Sherbeny, "Vehicle routing with time windows: An overview of exact, heuristic and metaheuristic methods," *J King Saud Univ Sci*, vol. 22, no. 3, pp. 123–131, 2010, doi: https://doi.org/10.1016/j.jksus.2010.03.002.

[26] I. N. Sutapa and I. G. A. Widyadana, "Studi Tentang Travelling Salesman dan Vehicle Routing Problem dengan Time Windows," *Jurnal Teknik Industri*, vol. 5, no. 2, pp. 81–89, Dec. 2003, doi: https://doi.org/10.9744/jti.5.2.81-89.

[27] N. A. El-Sherbeny, "Vehicle routing with time windows: An overview of exact, heuristic and metaheuristic methods," *J King Saud Univ Sci*, vol. 22, no. 3, pp. 123–131, 2010, doi: https://doi.org/10.1016/j.jksus.2010.03.002.

[28] N. Azi, M. Gendreau, and J. Y. Potvin, "An exact algorithm for a single-vehicle routing problem with time windows and multiple routes," *Eur J Oper Res*, vol. 178, no. 3, pp. 755–766, May 2007, doi: https://doi.org/10.1016/j.ejor.2006.02.019.

[29] R. Baldacci, A. Mingozzi, and R. Roberti, "Recent exact algorithms for solving the vehicle routing problem under capacity and time window constraints," *Eur J Oper Res*, vol. 218, no. 1, pp. 1–6, Apr. 2012, doi: https://doi.org/10.1016/j.ejor.2011.07.037.

[30] I. James, *Navigation and Nautical Astronomy: For the Use of British Seamen*, 3rd ed. The Board of Longitude, 1835.