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A Multi-Item Probabilistic Inventory Model that Considers Expiration Factor, All Unit Discount Policy and Warehouse Capacity Constraints

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Abstract: The characteristics considered in this study are probabilistic demand, perishable products, and warehouse constraints for multi-item inventory models. This condition occurs in several industries that consider perishable factors and warehouse constraints, namely companies that produce food, food sales agents, and retail that sell goods to end customers. The Karush-Kuhn-Tucker Condition approach was used to solve the warehouse capacity problem to find the optimum point of a constrained function. The results of the developed inventory model provide two optimal ordering times, namely ordering time-based on warehouse capacity and joint order time, and the two ordering time values will be compared to determine which ordering time is optimal. In addition, the sensitivity analysis to the model was done to analyse the total inventory costs in a planning horizon, the time between goods ordering from one cycle to the next cycle, and the number of items that will expire. The parameters to be changed for the sensitivity test were warehouse constraint, a fraction of good condition goods, holding costs per unit per period, and all unit discount factors. The sensitivity analysis was done to see the behaviour of the total cost, time to order changes, and the quantity of perished products. The result of model testing and sensitivity analysis showed that total cost, based on joint order, is sensitive to the fraction of good condition products, discount, and holding cost. The joint order was not sensitive to the warehouse capacity. In general, the model was perceived as able to describe the behaviour of the model components.

Keywords: Probabilistic inventory model, all unit discount, perishable, warehouse capacity

Introduction

Inventory can be defined as a stock of goods stored in a warehouse to meet future needs. These inventory forms can be classified as raw materials, work-in-process materials, finished goods, packages, and general supplies [1]. Generally, all companies always have inventory. In financial terms, inventory is an asset but also an expense. The existence of inventory can be a burden because it is a form of waste. Therefore, good inventory management is essential for a company because it is related to the balance between service levels on the one hand and costs on the other [2]. Criteria performance of inventory management can be defined by three aspects based on actors in system inventory. The three criteria are service level (availability and serviceability) and for consumers, total costs for management, and inventory turnover ratio for owners [3].

Small to medium-scale retailers that sell consumer products such as mini markets generally have two limitations [4].

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First, the products sold are generally dominated by food products with a short product life (generally, food products are perishable products). Second, there is a very significant limited storage space. Both limitations significantly affect the inventory system, especially determining the lot size for ordering goods in the replacement process. On the other hand, taking advantage of discounts offered by suppliers will generally encourage large order sizes. The impact of discount on inventory system is decreasing purchase price and holding cost per unit of product (generally holding cost (stock carrying cost) per unit is proportional to the price of the product) [2]. Therefore, retail company management such as minimarkets needs to optimize their inventory system by considering the trade-off between supplier discount offers and two existing limitations.

Research on inventory systems that consider factors such as expiry date, joint orders, and all unit discounts have been carried out, for example, [5, 6, 7,8], and [9]. Research on inventory control models that considers limited warehouse capacity on deterministic demand has been carried out by [10], while probabilistic demand has been carried out by [11] and [12]. Inventory models with probabilistic demand are considered more representative of actual conditions [13]. Here, to deal with the probabilistic demand, organizations usually prepare safety stock, which is determined directly from a forecast ([14, 15]).

From the studies of [10, 11], and [12] that have been done, all of them have not included the perishable factor as consideration for determining optimal ordering lot size. A study considers multi-item, space capacity, and perishable items but does not include discount, with the probabilistic nature presented in uniform distribution [16]. This study intended to develop an inventory model suitable for the retail industry by considering product types (multi-item), the nature of probabilistic demand, expired factor (perishable product), all-unit discount, and limited warehouse capacity.

Methods

his study aimed to develop a multi-item probabilistic inventory model that considers the expiration factor (perishability), the all-unit discount, and limited warehouse capacity. A multi-item probabilistic inventory model that considers expiry factors refers to [7] and [2]. The model developed in [7] is an EOQ inventory model that considers the expiration period for a multi-item inventory model. Meanwhile, Koswara and Lesmono [2] have developed a probabilistic inventory model that considers the all-unit discount and limited warehouse capacity to determine the optimal time between orders. The numerical data used in this study were taken from the research of [2] by adding expiration date data.

Inventory models that consider limited warehouse capacity will generally form a nonlinear function with constraints (nonlinear programming). The constraint function can be a linear or nonlinear function, either in an equation or an inequality. Two general approaches can solve this inventory model problem, i.e., a calculus-based optimization approach and a heuristic or metaheuristic algorithm-based optimization approach. First, solve nonlinear programming problems with a calculus optimization approach generally using the Karush-Kuhn-Tucker (KKT) Conditions or the Lagrange multiplier method. Second, a metaheuristic approach (such as a genetic algorithm, simultaneous annealing algorithm, ant colony algorithm, and others) can be chosen to solve nonlinear programming if the model being evaluated is included in the NP-Hard problem, or there is fast computation time required. The solution-solving method used in this study is the KKT Conditions method. The KKT Conditions method was chosen because this method is more suitable for solving nonlinear problems with constraints in the form of inequality. Meanwhile, the Lagrange Multiplier method, which is more suitable for solving nonlinear problems, is in equations [17].

The decision variable for a multi-item inventory model with a warehouse capacity limitation is the time between orders (replacement). There are two terms of time between orders. First, the time between orders is defined as result of dividing available warehouse capacity (in terms of volume or weight) and total demand volume (the result from multiplying total demand by volume or weight of each type of product), hereinafter referred to as T_{max}. Second, the time between orders is defined as time of joint orders, hereinafter referred to as the T_{opt}. The T_{opt} is obtained from the first derivative of the total cost function against T_{max} variable. If T_{opt} value is smaller than T_{max} value, decision variable is the T_{opt}. Conversely, if T_{opt} value is greater than T_{max}, then the T_{opt} solution violates the constraint function so that the optimal decision variable for the model is T_{max} value. The inventory policies involve order quantities and total cost inventory, which are based on these T_{max} and T_{opt} values.

This study carried out an inventory policies comparison between the proposed model and the models from [2] and [7]. Moreover, sensitivity tests were conducted on some parameters that affect the inventory policy trade-off. These parameters are warehouse capacity, discount factor, good goods fraction, and storage (holding) costs.

Model Assumptions

The assumptions used in this study are as follows:

1. All types of items are ordered from same supplier (joint order) and have same lead time (L).
2. The good fraction value (θi) is 90% for all types of items.
3. The existence of expired items has consequences on two cost components, namely cost of shortages and cost of expiration:
   a. Consequences on shortage costs: the existence of expired goods causes reduced availability of goods to meet all demand, so that there is a demand that cannot be fulfilled.
   b. Consequences on expiry costs: items entering expiry date will be sold at a lower price than purchase price, resulting in a loss equal to the gap between purchase cost and selling price items (on expired date)
4. All expired items will be sold at the end of period t_{1_i} simultaneously so that there are no expired items left during period t_{2_i}.
5. All expired items can still be sold to certain parties at J_i price (where $J_i < P_i$). This assumption means that expired items still can be used (sold), but not for consumption (food).
6. Volume size of each item is known with certainty at the beginning of a planning period.
7. Stockout items as lost sales
8. The lost sales cost is opportunity cost, where the amount is equal to profit of each type of product.
9. Lost sales costs due to probabilistic demand and expired items are the same.

Notation

The following is notation used:

D_i : Total demand for item i in one planning horizon. (unit/year)
P_{li} : Purchase price item i per unit for each category lot l ($/unit)
S_i : Demand standard deviation item i during planning horizon (Rp/unit/year)
H_i : Holding cost items i for one planning horizon ($/unit/year)
c_{u_i} : Lost sales (stock out) cost for items i (Rp/unit)
J_i : Selling price per unit of items i that will expire (Rp/unit)
n : Numbers of item types (unitless)
T^* : The optimal joint order (year)
Q_i : Order size optimal for item i (unit)
Q_{k_i} : Number expired items for item i (unit)
α : probabilities stock out (shortages) of supplies
$Z_{\alpha i}$: Z value for each item i in normal distribution at level α
$f_{z_{\alpha_i}}$: Ordinate function value for item i based on Z value
$\varphi_{z_{\alpha_i}}$: Partial expectations for item i
N_i : Shortages number expected of each item i (unit)
N_{T_i} : Shortages number expected for item i during one planning horizon (unit)
ss_i : Number of safety stock for item i (unit)
t_i : Short cycle planning period in one horizon planning (year)
t_{1_i} : The length of keeping period for item i until just before enters expiration date (year)
t_{2_i} : The length of shortage period for item i (year).
θ_i : Good condition fraction for item i (0<θ_i<1)
U_{li} : Quantities order requirement for item i in an interval price break l to get for all discount condition
$1-\theta_i$: Expired condition fraction for item i (0<$1-\theta_i$<1)
L : Lead time order (year)
W : Total warehouse capacities (unit of volume)
w_i : Volume size for item i (unit of volume)
O_b : Total purchase cost for during one planning horizon (Rp)
O_p : Total ordering cost for during one planning horizon (Rp)
O_s : Total holding cost for during one planning horizon (Rp)
O_k : Total lost sales (shortage) cost for during one planning horizon (Rp)
O_{kd} : Total expired cost for during one planning horizon (Rp)
O_T : Total inventory cost for during one planning horizon (Rp)

Model Formulation

The inventory model in this study results from developing a probabilistic inventory model by considering expiration factor and purchase discount factors using all unit discounts [7] with the additional aspects from multi-item and limitations of warehouses capacities [2]. Demand's nature is probabilistic is described by demand's average level for item i is D_i and the demand's standard deviation for item i is S_i. The maximum inventory level for item i is Q_i^*. Where the length of time between orders $\left(T^* = \frac{D_1}{Q_1^*} = \frac{D_2}{Q_2^*} = \frac{D_i}{Q_i^*}\right)$ for all items i is the same. This condition can occur because it is assumed that all items are ordered from the same supplier and at the same time. Therefore $Q_i^* = T^*D$.

Inventory shortage conditions in this inventory mode are caused by two factors: the first is due to expired goods; the second is due to the uncertainty of demand (demand's nature is probabilistic). Inventory shortage condition for each item i occurs at time t_{2i}. Level inventory illustration in this research for one's cycle replacement is as Figure 1.

By using the similarity approach, the equation of t is written as follow:

$$\frac{Q_i}{T^*} = \frac{(Q_i - Q_{ki})}{t_{1i}}$$

$$t_{1i} = \frac{T^*(Q_i - Q_{ki})}{Q_i}$$

whare $Q_{ki} = (1-\theta)_i Q_i$ and $\theta_i = \frac{(Q_i - Q_{ki})}{Q_i}$,

so $t_{1i} = T^*\,\theta_i$

$T^* = t_{1i} + t_{2i}$, so $t_{2i} = (1-\theta_i)T^*$

Because the expired condition fraction is known at the beginning of the planning period, so the average inventory shortage due to expired goods is expressed as equation 1 as follows;

$$\frac{Q_{ki}}{2} = \frac{Q_i^*(1-\theta_i)}{2} = \frac{T^*D_i(1-\theta_i)}{2} \quad (1)$$

Where condition expired occurs at time T1, then equation 1 becomes;

$$\frac{T^{*2}D_i(1-\theta_i\hat{\ })^2}{2} \quad (2)$$

The level of average expired items for item i in one planning horizon becomes;

$$\frac{T^*D_i(1-\theta_i\hat{\ })^2}{2} \quad (3)$$

The expected value of inventory shortage due to demand's nature is probabilistic, showed by $E[N]$.

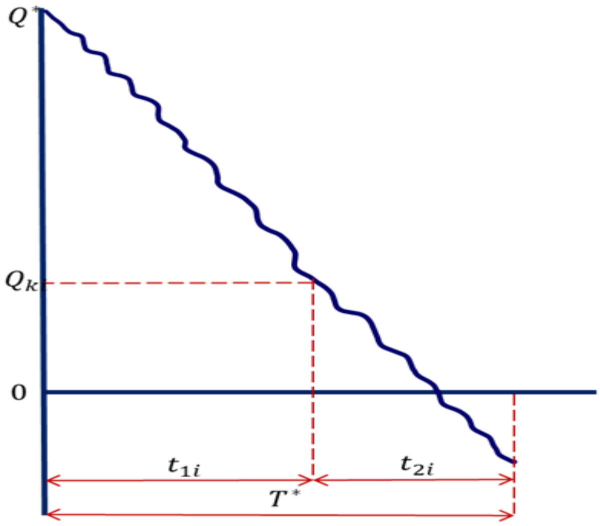

Figure 1. Illustration of inventory position in one cycle

Demand's probability distribution and inventory shortage's probability distribution is assumed to be the normal distribution, so the equation for expecting shortages base on [3] is as follows

$$E[N_i] = S_i\sqrt{L}(f_{z\alpha} - Z_\alpha\varphi_\alpha) \tag{4}$$

Total inventory shortage for one planning horizon caused of demand's nature is as

$$\frac{E[N_i]}{T^*} = \frac{S_i\sqrt{L}(f_{z\alpha}-Z_\alpha\varphi_\alpha)}{T^*} \tag{5}$$

There are 5 components of total inventory cost in this research, namely purchase costs (O_b), ordering costs (O_p), holding (carrying) costs (O_s), lost sales (shortage) costs (O_k), and expiry costs (O_{kd}), which can be arranged in the following equation

$$O_T = O_b + O_p + O_s + O_k + O_{kd} \tag{6}$$

In this study, the all-unit discount policy applies. Purchase price will be adjusted according to the order lot size. The purchase cost is multiplication of number of goods for each type purchased (Q_i) with prevailing price according to order lot size of each items (P_{li}). The equation is expressed as

$$O_b = \sum_{i=1}^{n} P_{li}D_i \tag{7}$$

Where P_{li}:

$$P_{li} = \begin{cases} a_{0i}, if\ U_{0i} \le Q < U_{1i} \\ a_{1i}, if\ U_{1i} \le Q < U_{2i} \\ \vdots \\ a_{ji}, if\ U_{ji} \le Q < U_{(j+1)i} \end{cases}$$

The joint order assumption means that all types of goods are ordered in one order ([6] and [18]). So, it is assumed that suppliers do not have limitations to fulfill the demand for each type of goods. Ordering cost for all items in one cycle period is A. Total ordering cost in one planning horizon is the multiplication of ordering cost per order and ordering frequency. However, it can also be calculated from ordering cost per order (A) divided by the optimal joint order time (T^*). The equation is expressed as

$$O_p = \frac{A}{T^*} \tag{8}$$

Holding cost per unit (H_i) of each item is proportional to its price. Where H_i is the multiplication of holding proportion cost per unit of items with price its item (P_{li}). Holding cost in one cycle is multiplication from holding cost per unit per period, average item stored, and length of stored time (t_{1_i}). Total holding cost for one planning horizon is holding cost per a cycle time multiplied by number of cycles, or by $1/T^*$. Hence holding cost equation can be written as follows

$$O_s = \sum_{i=1}^{n}\left(H_i \times \frac{1}{2}(T^*D_i + T^*D_i(1-\theta_i) \times \theta_i\right) + \sum_{i=1}^{n} H_i ss_i$$
$$O_s = \sum_{i=1}^{n}\left(\frac{H_i\theta_i(T^*D_i+T^*D_i(1-\theta_i))}{2} + H_i ss_i\right)$$
$$O_s = \sum_{i=1}^{n}\frac{H_i(T^*D_i\theta_i+T^*D_i\theta_i(1-\theta_i)+2ss_i)}{2} \tag{9}$$

Here, safety stock is estimated using the probability of shortage (α). The safety stock $ss_i = Z_\alpha S\sqrt{L}$ is the estimated safety reserve for one planning horizon. The number of safety stocks will affect holding cost and reorder point but not affect the time's length between order cycles (T optimal).

Total number of shortages is influenced by probabilistic demand factors and expiration factors. The total shortage (lost sales) cost is the multiplication of the shortage cost per item type (cu_i). Total level of inventory shortage in one planning horizon is the sum of the average items expired and expected shortages (equation 3 and equation 5). Total cost of lost sales can be expressed as bellow

$$O_k = \sum_{i=1}^{n}\left(\frac{cu_iD_i(1-\theta_i)^2{T^*}^2+2E[N_i]}{2T^*}\right) \tag{10}$$

Shortage expectations are assumed to have a normal distribution with the shortage probability value set at the beginning of the planning period (equivalent to 1 minus value of expected service level probability). Numbers of shortage expectations for one cycle replenishment expressed as $E(N) = S\sqrt{L}(f_{z_\alpha} - Z_\alpha\psi_{Z\alpha})$. Total number of shortages for one horizon planning expressed as $\frac{E(N)}{T^*}$.

Inventory control that considers expiration factors can be found in general retail industry, food industry (manufacturing and restaurant), fast manufacturing consumer goods (FMCG), and chemical and pharmaceutical industries. In this developed model, the items enter their expiration date at t_2 (after passing through t_{1_i}). Assuming expired fraction is known, so period t_2 is also known. Level of expired item for one cycle replenishment express as $Q_{ki} = (1-\theta_i)Q_i^* = (1-\theta_i)T^*D_i$; and Q_{ki} for one planning horizon become $(1-\theta_i)D_i$. All expired items can be sold to other parties with low price (J_i), which is the price of an expired item less than the purchase cost. Hence the cost of one item expired is the gap between purchase cost and the price sold of an expired item. Total expiry cost is multiplication of number item i was expired in one cycle, expiration cost $(P_i - J_i)$ item i, and total cycles number in one planning horizon.

$$O_{kd} = \sum_{i=1}^{n} D_i(1-\theta_i) \times (P_i - J_i)$$

Since the safety stock for one planning horizon also can be expired, the cost becomes

$$O_{kd} = \sum_{i=1}^{n} (D_i + ss_i)(1-\theta_i) \times (P_i - J_i) \tag{11}$$

Total inventory cost in one planning horizon is expressed as

$$O_T = \sum_{i=1}^{n} P_{li}D_i + \frac{A}{T^*} + \sum_{i=1}^{n}\left(\frac{H_i(T^*D_i\theta_i + T^*D_i\theta_i(1-\theta_i) + 2ss_i)}{2}\right) + \sum_{i=1}^{n}\left(\frac{cu_i(D_i(1-\theta_i)^2{T^*}^2 + 2E(N_i)}{2T^*}\right) + \sum_{i=1}^{n}(D_i + ss_i)(1-\theta_i) \times (P_i - J_i) \tag{12}$$

Warehouse capacity constraint is expressed as inequality function, is as follow

$$\sum_{i=1}^{n} w_iQ_i \le W \tag{13}$$

Because decision variable of model is as T^*, equation (13) can be expressed as

$$g(T) = \sum_{i=1}^{n} w_iD_iT_{max} \le W \tag{14}$$

The Karush-Kuhn-Tucker Conditions for this non-linear programming is as follow

$$Min\ O_T(T_{max})$$
$$\text{s/t } g(T_{max}) \le W \tag{15}$$

Hence

$$T_{max} = \frac{W}{\sum_{i=1}^{n} w_iD_i} \tag{16}$$

$$T_{opt} < \sqrt{\frac{2(A + \sum_{i=1}^{n} cu_iE[N]_i)}{\sum_{i=1}^{n}\frac{\left(H_iD_i\theta_i(2-\theta_i) + cu_iD_i(1-\theta_i)^2\right)}{2}}} \tag{17}$$

T_{opt} in equation (17) is also the optimal solution for $\frac{\partial O_T}{\partial T} = 0$, the first derivative of equation (12). Furthermore, T_{opt} is called time between joint orders. The optimal solution for minimizing total costs inventory with warehouse capacity constraints is T^*. According to the first theorem of Karush-Kuhn-Tucker Conditions, the value of T^* is T_{opt} if $T_{opt} < T_{max}$, and it is T_{max} if $T_{opt} > T_{max}$ (this condition occurs when $g(T_{max}) - W > 0$, so this inequity must be transformed to be an equation $g(T_{ma}) - W = 0, \lambda > 0$).

Due to the consideration of all unit discount and multi product aspect, the algorithm for determining the inventory policy (time between orders and total inventory cost) in this study is as follows:
Calculate T_{max} value using equation (16).
Calculate T_{opt} using equation (17).
If $T_{max} \ge T_{opt}$, the solution of the inventory policy is T_{opt} $(T^* = T_{opt})$, otherwise the solution is $T_{max}(T^* = T_{max})$.
Determine price (P_{li}) base on T^* value (point 3): convert T^* value to order lot size Q_i^* $(Q_i^* = T^*D_i)$, and order lot size will determine the pricing policy (if $Q_i^*$is in the range of price break l then use the price p_{li}).
Calculate total inventory cost using equation (12).

Results and Discussions

Data and Calculation

Numerical data set used for model testing was taken from Limanjaya and Silitonga [7] with an additional expiration factor. The data can be seen in Table 1 and Table 2.

Based on the input data from Table 1 and Table 2, it is obtained that $T_{opt} > T_{max}$. If $T_{opt} > T_{max}$ interpret then the objective function solution, which minimized the total inventory cost (first derivative from total cost function), is hindered by the warehouse capacity constraint. Hence, the solution of time between orders is the value of T_{max}(0,1429 years = 52.2 days, 365 days in a year) see Table 3.

Total cost of using T_{max} value is indeed smaller than total cost using T_{opt} value. To achieve optimal costs, warehouse capacity constraints must be relaxed. Initial warehouse capacity is 500 units of volume, and it must be relaxed to 1516,75 units of volume if we want to get the optimal cost.

Analysis

Comparative analysis was performed on the following inventory models:

1.

2. Limanjaya and Silitonga's model is the first model [7]: a multi-item probabilistic inventory model that considers expiration factors and purchase bonuses. To make a more representative comparison, the purchase bonus policy was replaced with all unit discounts policy.

The second model is Koswara and Lesmono's model [2]: a multi-item probabilistic inventory model, with all unit discount policy and warehouse capacity cons traints. Demand during the lead time using gamma distribution was replaced with normal distribution to be more representative to be compared with this study. It should be noted that there is no expired cost in this model.

Table 1. Data of product (numerical set data)

Description	Item 1	Item 2	Item 3
Total demand expectation D_i (unit/year)	550	400	800
Volume size w_i (unit of volume)	2	2	2
Demand standard deviation S_i (unit/year)	57.75	42	72
Lead time L_i (year)	0.0083102	0.0083102	0.0083102
Shortage expectation value N_i	0.3596	0.3046	0.6028
Fraction good condition θ_i	0.90	0,90	0.90
α_i Value	5%	6%	7%
Z- Value	1.65	1.55	1.45
f_{z_α} Value	0.1023	0.12	0.1394
φ_{z_α} Value	0.0206	0.0261	0.0328

Table 2. Data of cost components (numerical set data)

Cost description	Item 1		Item 2		Item 3	
Price component ($/unit)	$Q < 201$	12	$Q < 131$	15	$Q < 301$	8
	$Q \geq 201$	10,5	$Q \geq 131$	13	$Q \geq 301$	7
Join order cost A ($)			9			
Holding cost per unit per period H_i ($/year)	0.12		0.225		0.08	
Shortage cost cu_i ($/unit)	3		4		2	
expired item price ($/unit)	10.2		12.75		6.8	

Table 3. Result.

	Value of T	
	T_{max}	$Topt$
Time between order	$0{,}1429\ year$	$0{,}43336\ year$
Total Cost Inventory	$ 19.371,31	$ 16.938,63

This study's third inventory model is a multi-item probabilistic inventory model that considers expiration factors, all unit discount policies, and warehouse capacity constraints.

All three models were tested using the same data set with their respective adjustments. The output of the three models can be seen in Table 4. The T^*-value from multi-item probabilistic inventory model by considering perishable and purchase bonus factors [7] is greater than the other two, whereas the multi-item probabilistic inventory model by considering all unit discount and limited warehouse capacity [2] and the model in this study have the same value. Basically, the T_{opt} value from model [2] and this study is almost the same as T value from model [7], but due to the limited warehouse capacity, the T^* optimal value used the T_{max}value from the equation (16). Model [7] provides lower total cost than the other two. This condition occurred because of the order lot size from the model [7] has the advantage of discounted price due to the absence of capacity constraint. Meanwhile, the models [2] and this research were not. Total inventory cost in this study is greater than the total cost of model [2], even though both models have the same decision variables (T* value, order lot size and order frequency). This is due to the expiration factor in the study.

The output from the numerical test is closeout mores complex of inventory models (consider many factors or element systems), the higher the total inventory cost will be. Besides that, optimal solutions are getting harder to achieve.

Sensitivity analysis has been conducted to determine the sensitivity of T_{max}, T_{opt}, Q_k and total costs to the changes of warehouse capacity, good goods fraction, discount factors, and holding cost. The effect from changing parameter values can be seen in Figure 2 up to Figure 5.

The value of $T_{max}$is sensitive to the warehouse capacity, good goods fraction, and holding cost. Increasing the capacity of warehouse will increase T_{max} value so that the order lot size becomes larger, and the number of ordering frequencies decreases. The bigger the order lot size, the bigger the discount will be. However, it will increase the total holding cost and the number of expired items. In this case, there is a trade-off between total purchase cost and total ordering cost against total holding costs and expiration costs. Total cost is sensitive to the good goods fraction, but theT_{max} value is not. The higher the fraction of good goods, will make the number of expired goods fewer, thus reducing total expired cost and total shortages (lost sale) cost. The value of T_{max} is not sensitive to the discount factor value, due to the capacity constraints, because the discount factor will only give effect to the T_{max} value if the value of capacity limitation is high enough.

The parameters of good goods fraction and holding cost are sensitive to change the total cost thru the value of T_{opt}. The total cost is also sensitive to the discount factor. The effect of the changes in good goods fraction to the total cost is the same, whether using T_{opt} value or T_{max} value. However, T_{opt} is sensitive to the changes of good goods fraction parameter. A decrease in good goods fraction will decrease T_{opt} as much as the percentage decrease in value of good goods fraction. Meanwhile, an increase in value of good goods fraction will increase T_{opt} value to a certain extent. The T_{opt} is no longer sensitive to the changes in value of good goods fraction when the percentage change approaches 1. The lower holding cost, the higher T_{opt} value, as the order lot size increases and ordering frequency decreases, vice versa. A 10% increase in holding cost will decrease T_{opt} value by 3-5%. Hence, T_{opt} is not sensitive to the changes in parameter value of warehouse capacity

The number of expired goods is sensitive to the value changes of good goods fraction parameter, either based on T_{max} value or T_{opt} value. The higher good goods fraction, the smaller the number of expired goods. The number of expired goods using T_{opt} value is greater than using T_{max} value. The number of expired goods becomes insensitive when increasing the percentage changes in good goods fraction, greater than 20%. The number of expired goods using T_{max} value is sensitive to the changes of warehouse capacity, but not sensitive if using T_{opt}. An increase in warehouse capacity by 20% will increase 20-22% of the number of expired items, and vice versa. The number of expired items using T_{opt} value is sensitive to the changes in parameter value of holding cost, but not sensitive if using T_{max} value. An increase in holding cost by 20% will decrease the number of expired items by 7-10%, and vice versa. The number of expired goods is not sensitive to the changes in discount factor, either based on T_{max} value or T_{opt} value.

For companies with deteriorating product characteristics, the developed model in this study is suitable. Efforts to reduce the total cost of an inventory system that has the same characteristics as this study can be made in several ways, including:

Minimizing the holding cost (carrying cost) per item. This can be done by managing inventory management properly so that the elements forming holding cost per item can be reduced. These elements can be in the form of labor, utilities (use of water and electricity), material handling, racking/shelving management, maintenance, and quality control.

In the existing condition, the good goods fraction cannot be controlled. However, the company can manage the purchasing process so that it obtains goods that still have a long expiration date. In addition, companies can manage inventory flow with the FEFO (First Expired First Out) system. Minimizing the number of expired items will also reduce the overall total cost.

Efforts to reduce total inventory cost can also be made by relaxing inventory system constraints. Factors that become inventory system constraints can come from financial capacity and resource availability, one of which is storage capacity. Relaxing the capacity constraints will increase order lot size. Thus, companies can take advantage of economies of scale of ordering and the opportunity to get discounted prices. The larger the order lot size, the less likely there is a shortage of inventory, but it will increase the expiration date and total cost of holding. This trade-off needs to be adequately considered by the company.

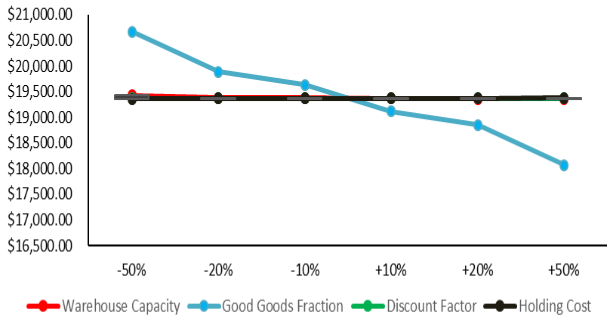

Figure 2. Total cost sensitivity analysis using T_{max}

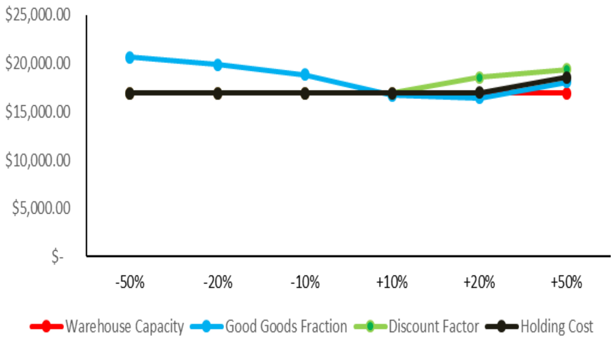

Figure 3. Total cost sensitivity analysis using T_{opt}

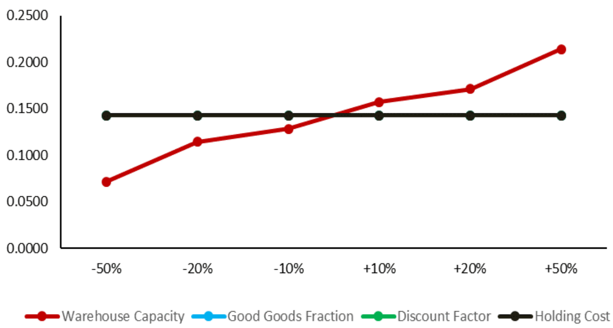

Figure 4. T_{max} sensitivity analysis

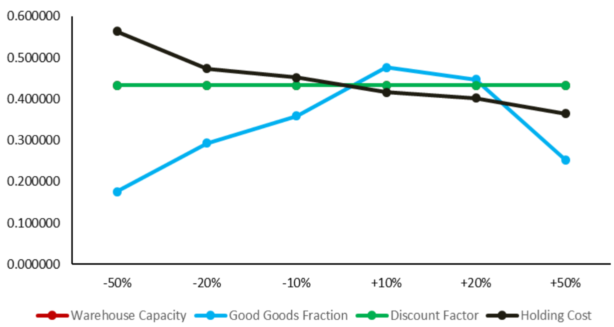

Figure 5. T_{opt} sensitivity analysis

Conclusion

In this study, a multi-item probabilistic inventory model that considers expiration factors, all unit discount policy, and warehouse capacity constraints have been developed. This inventory model can be applied to industries with expired product characteristics (gradually deteriorating over time). Every company with limited resources needs to pay attention when managing an inventory system. This study takes the limiting factor of storage capacity in modeling the inventory.

By looking at the results of comparative model analysis, it can be concluded that an inventory model that considers more factors will have a greater total inventory cost than the one that considers fewer. The results from sensitivity analysis concluded that T_{max} value is very sensitive to warehouse capacity limitation parameter, while T_{opt} value is very sensitive to the good goods fraction and holding cost. The trade-off between order lot size against capacity and holding cost is the most critical factor in determining the inventory policy.

Future studies can consider different expiration factors for each type of goods, i.e., the expiration period for each product is different from each other. Future studies can also consider expanding the warehouse capacity by considering the availability of funds. In actual conditions, storage capacity can generally be relaxed, but this relaxation ability is very dependent on financial constraints.

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