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The Development of Order Quantity Optimization Model for Growing Item Considering the Imperfect Quality and Incremental Discount in Three Echelon Supply Chain

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Abstract: This research develops an optimization model for determining the order quantity for growing items by considering the imperfect quality and incremental discount by involving three supply chain members: farmers, processors, and retailers. The farmers are responsible for caring for the newborn items until they reach their ready-to-eat weight. The processors perform two roles, namely processing and screening. In the processing role, the processors process the grown items by a slaughtering and packaging process. Afterward, they inspected the processed items and categorized the items into good and poor quality. Finally, they shipped the end products to retailers. The retailers are responsible for selling good-quality items to the final consumers. This research considers two kinds of poor quality. First is the poor quality of growing items in terms of mortality rate. The second is the poor quality of final products on the processor side. The processed items with poor quality are then sold to the secondary market at lower prices in one batch at the end of the period. This model also considers the incremental discounts offered by vendors to farmers and retailers to consumers for specific amounts of purchases. The model's objective function is to maximize the total supply chain profit, with the number of orders quantity, cycle time, and the number of batches delivery set as the decision variables. The sensitivity analysis results show that the most sensitive parameter in the model is the probability that the live items survive throughout the growth period.

Keywords: Growing item, economic order quantity, imperfect quality, incremental discount, inventory system, supply chain.

Introduction

Inventory control plays an essential role in the production system. The company hedges demand for the most profitable products. Inventory control is one of the most critical factors that help reduce costs or increase profits [1]. Therefore, inventory control is a significant field for real-world applications and research purposes. In the inventory model, uncertainty is treated as randomness and handled using probability theory. The most widely used inventory model is the Economic Order Quantity (EOQ) model, where continuous operations are classified as supply and demand [2].

Mishra [3] explained that the development of inventory modeling began in the second decade of the 19th century when Harris introduced the inventory model. Then, in 1934 Wilson developed it by deriving a mathematical model to get the economic order quantity. This model is widely known as the classic EOQ. However, the application of the Harris model has some practical limitations due to the assumptions, such as it only can be used to solve the inventtory item where the item undergoes no physical changes during the planning period. This assumption is one of the drawbacks since some items experience material changes during the planning period, such as improvements, declines in quality, and growth.

Rezaei [4] is the first researcher who proposed the optimal order quantity model for the growing item. Growing items are items that experience growth over time continuously during the storage period before they can finally be sold or consumed. The increase in weight on the items during the growth period is the difference between growing items and conventional items. Zhang *et al.* [5] proposed a model with growing items taking into account the carbon tax. Nobil *et al.* [6] proposed a model with growing items taking into account the inventory shortages. Sebatjane and Adetunji [7] proposed a growing items model with imperfect quality. Then Sebatjane and Adetunji [8] proposed a growing items model considering the incremental discounts. Hidayat *et al.* [9] proposed an optimization model for growing items with incremental quantity discounts, capacitated storage facility, and limited budget. Luluah *et al.* [10] proposed a model for growing items with incremental discount and imperfect quality on the farmer side. Furthermore, Sebatjane and Adetunji [11] proposed a growing items model by considering imperfect quality in a four-echelon supply chain.

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In purchasing newborn growing items, vendors often offer incremental discounts where the vendor provides a reduction in the purchase price if the number of items purchased by the company is within a *cut-off* of a specific quantity range that has been set (Sebatjane and Adetunji [8]). Incremental discounts offered by the vendor aim to encourage buyers to buy products in larger quantities. In addition, the existence of incremental discounts offered by the vendor makes the company make the right decision concerning the optimal purchasing quantity to maximize the profit.

Sebatjane and Adetunji [11] conducted a study to develop a model in determining the optimal order for growing items in a four-echelon supply chain by considering the imperfect quality. The study considered a mortality rate during the growth of the items due to diseases, pests (in the case of plants), and predators (in the case of livestock). Then, the growing items that are ready to be consumed are generally rare in their original form. Therefore, they must be processed before they are ready to be consumed. Thus, the items undergo several stages until it reaches a form suitable for consumption. The supply chain consists of four stages, namely, farming, processing, screening, and selling. The research has not considered the incremental discount offered by the seller to the buyer.

This study developed an optimization model in determining the optimal order quantity for growing items by considering the imperfect quality and incre-mental discounts in a three-echelon supply chain (i.e., farmers, processors, and retailers). In this research, we develop the EOQ model in supply chain optimization for growing items considering the imperfect quality and incremental discount based on the research of Sebatjane and Adetunji [8] and [11]. We extend those researches by adding imperfect quality and incremental quantity discounts in purchasing newborn items from vendors to farmers and final products from retailers to consumers for specific amounts of purchases. The model's objective function in this research is to maximize the company's profit with the decision variables in order quantity, order cycle time, and the number of delivery batches.

Methods

Assumptions

The following assumptions are used in this research: (1) The model considers three members of supply chain, consisting of one farmer, one processor, and one retailer selling one type of growing item. (2) There are some growing items that not survive until the end of the growth period. (3) Growing items do not have the ability to reproduce during the growth cycle. (4) Some of the growing items do not meet the required quality standard and are classified as poor quality. (5) Processing rate (R) and screening rate (s) are greater than demand rate (D). (6) The good quality products will be sent to the retailer and poor-quality products will be sold to the secondary market at lower price.

Notations

The objective function of this model is to maximize the total profit with three decision variables.

Y : The optimal order quantity (unit)
T : Cycle time (week)
n : The number of batches delivery (time)

The parameters in this model are as follows:

D : Demand rate for good quality processed items in weight units per unit time (kg/week)
R : Processing rate in weight units per unit time (kg/week)
$w(t)$: The weight of an item at time t (kg)
w_0 : Newborn weight of each item (kg)
w_1 : Target weight of each item (kg)
x : Probability of the live items survive throughout the growth period
P^v : Procurement (or purchasing) cost per weight unit of live newborn item (IDR /kg)
R^v : The sum of the terms in P_v which are independent of Y (IDR /kg)
K^f : Farmer's setup cost per cycle (IDR)
c^f : Farmer's feeding cost per weight unit (of live inventory) per unit time (IDR /kg/week)
m^f : Farmer's mortality cost per weight unit (of dead inventory) per unit time (IDR /kg/week)
T^f : Duration of the farmer's growth period (week)
P^f : Farmer's selling price per weight unit of live items (IDR /kg)
K^p : Processor's processing facility setup cost per cycle (IDR)
h^p : Processor's processing facility holding cost per weight unit per unit time (IDR /kg/week)
T^p : Time required to process the entire lot size (week)
K^s : Processor's cost of sending a single batch of good quality processed inventory to the retailer (from the screening facility) (IDR)
h^s : Processor's screening facility holding cost per weight unit per unit time (IDR /kg/week)
τ : Time between consecutive deliveries of good quality batches from the screening facility to the retailer (week)
n : Number of batches of good quality processed inventory delivered to the retailer during a single screening run
T^s : Time required to screen the entire lot size (week)
P^p : Processor's selling price per weight unit of good quality inventory (IDR/kg)
P^q : rocessor's selling price per weight unit of poorer quality inventory (IDR/kg)

a	:	Probability of processed inventory that are of poorer quality
s	:	Screening rate in weight units per unit time (kg/week)
l	:	Screening cost per weight unit (IDR)
z	:	Number of items per batch of good quality processed items sent (by the processor) to the retailer
T	:	Replenishment cycle time for all echelons (week)
K^r	:	Retailer's ordering cost per cycle (IDR)
h^r	:	Retailer's holding cost per weight unit per unit time (IDR/kg/week)
P^r	:	Retailer's selling price per weight unit of good quality processed inventory (IDR/kg)
R^r	:	The sum of the terms in P_r which are independent of Y (IDR /kg)
α	:	The items' asymptotic weight (kg)
β	:	Constant of integration
λ	:	Exponential growth rate of the items (/week)

Model Formulation

The model formulation will be described using the farmers, processors, and retailers' perspective, where each transaction is offered with an incremental discount.

Farmer's Profit

The farmer purchases a new growing item from the vendor with initial weight (w_0), then he or she takes care for and feed until reaches a specified weight and stated as ready for consumption (w_1) during the growth period (T_f). In purchasing the new growing items, the vendor offers incremental discounts. In the Sebatjane and Adetunji [7], the vendor provides a reduction in the purchase price if the number of items purchased by the farmer is above a certain *cut-off*.

$$P_j = \begin{cases} P_1, y_1 = 0 \leq Y < y_2 \\ P_2, y_2 \leq Y < y_3 \\ \vdots \quad \vdots \\ P_j, \ y_m \ \leq Y \end{cases} \quad (1)$$

This incremental discount offer causes the purchase price to be lower if the number of purchases increases $P_1 > P_2 > \cdots > P_j > P_{j+1}$. So that the purchase costs in farmer side are as follow:

$$PC_f = P_1\left(Y_2 - Y_1\right) w_0 + P_2\left(Y_3 - Y_2\right) w_0 + \cdots + P_{v-1}\left(Y_v - Y_{v-1}\right) w_0 + P_v\left(Y - Y_v\right) w_0 \quad (2)$$

where R^v can also be rewritten as:

$$R^v = \begin{cases} \sum_{i=1}^{v} P_i(Y_{i+1} - Y_i)w_o, v \geq 2 \\ 0 \qquad\qquad\qquad\qquad v = 1 \end{cases} \quad (3)$$

So that the total purchasing cost of the farmer can be calculated by the following equation:

$$PC^f = R^v + P^v\, w_0\,(Y - Y_v) \quad (4)$$

The farmer's *setup* cost is related to the arrangement for a new growing items cycle and set as a *fixed cost*. The farmer's *setup* cost can be calculated as follows:

$$\mathrm{K\,C}^f = K^f \quad (5)$$

Growing items that have been purchased are then maintained and allowed to grow until the weight reaches a specified target (w_1). At that time, the item will be sent to the processor for processing and screening. The growth function is assumed to follow an S-shaped curve (*sigmoid curve*) as shown in Equation (6).

$$w(t) = \frac{\alpha}{1+\beta e^{-\lambda t}} \quad (6)$$

When growing period ends, the surviving item's weight will reach the target w_1. Hence, the length of the growth period (T_f) can be calculated by Equation (7).

$$T^f = \frac{\ln[\frac{1}{\beta}(\frac{\alpha}{w_1} - 1)]}{\lambda} \quad (7)$$

During the growing period, some of the growing items can't survive. The probability of surviving items ($E[x]$) is assumed to follow uniform distribution. of random variables with a probability *density function*. The feeding cost of growing items during the growing period is calculated as follow:

$$\mathrm{FC}^f = c^f\ x \int_0^{T^f} y\, w(t)\, dt$$
$$= c^f\ x\, Y\, \{\alpha\, T^f + \frac{\alpha}{\lambda}[\ln(1 + \beta e^{-\lambda\, T^f}) - \ln(1 + \beta)]\} \quad (8)$$

The farmer incurs a cost associated with disposing the fraction of newborn items which do not survive until the end of the growing cycle. The farmer's mortality cost cycle (MC_f) is determined as the product of the farmer's average inventory level the fraction of items which do not survive 1- $E[x]$, so the mortality cost is calculated by the Equation (9).

$$\mathrm{MC}^f = m^f(1 - x) \int_0^{T^f} y\, w(t)\, dt$$
$$= m^f\ (1 - x)\, Y\, \{\alpha\, T^f + \frac{\alpha}{\lambda}[\ln(1 + \beta e^{-\lambda\, T^f}) - \ln(1 + \beta)]\} \quad (9)$$

When the growing cycle of the farmer ends, some items have died, so the total weight of the surviving items. Then the items will be sent to the processor at once. The farmer will receive a payment from the processor with the price of P_f for each weight unit of the surviving items. The farmer revenue (TR_f) can be calculated by the Equation (10).

$$TR^f = P^f\ Y\ x\ w_1 \quad (10)$$

The total profit of the farmer per cycle (TP_f) is the revenue of the farmer per cycle minus the total cost of the farmer per cycle so as to produce the following equation:

$$TP^f = P^f\, Y\, x\, w_1 - R^v + P^v\, w_0\, (Y - Y_v) - K^f - Y\left(c^f\, x + m^f\,(1 - x)\right)\left\{\alpha\, T^f + \frac{\alpha}{\lambda}[\ln(1 + \beta e^{-\lambda\, T^f}) - \ln(1 + \beta)]\right\} \tag{11}$$

Processor's Profit

The processor has two facilities, namely processing and screening.

Processing Facility

All growing items that survive until the end of the period at the farmer are accepted by the processor. The items will be processed by the processor to result a product that are ready to be sold to final consumers.

The processing is at a rate R, so the duration of the processing time can be determined by:

$$T^p = \frac{Y\, x\, w_1}{R} \tag{12}$$

The processor incurs a fixed cost at the start of each process, therefore the setup cost per cycle is:

$$\mathrm{KC}^p = K^p \tag{13}$$

The cost of purchasing of the processing (PC_p) is the same as the amount of sales of farmers, namely:

$$PC^p = TR^f = P^f\, Y\, x\, w_1 \tag{14}$$

The holding cost per processing cycle (HC_p) is found by multiplying the unit storage cost per weight of item (h_p) as shown in Equation (15):

$$\mathrm{HC}^p = h^p \left(\frac{Y^2\, x^2\, w_1{}^2}{2\, R}\right) \tag{15}$$

The total processing cost is the sum of setup cost, purchase cost, and storage cost as shown in the following equation:

$$\mathrm{TC}^p = K^p + Y\, P^f\, x\, w_1 + h^p \left(\frac{Y^2\, x^2\, w_1{}^2}{2\, R}\right) \tag{16}$$

Screening Facility

All items that have been processed will be transferred to the warehouse after the screening process to ensure its quality. At the screening process, there will be a holding cost, sorting cost, and shipping cost to retailers. Activities of sorting are directed to determine the poor-quality product. Hence, the weight of good quality product inventory is $Y\, x\, w_1(1 - a)$. The screening will ship n batches with each weight to the retailer in equal time intervals. All the processed items will be sorted at a rate of s, so the duration of the entire sorting time (T_s) can be calculated by:

$$T^s = \frac{x\, Y\, w_1}{s} \tag{17}$$

During the duration of the entire screening period, the processor sends a number of shipments (in this case n) at equal time intervals of good quality processed inventory to the retailer. Therefore, the time interval between deliveries of good quality products is:

$$\tau = \frac{x\, Y\, w_1}{s\, n} \tag{18}$$

The screening sends n batch of good quality processed inventory, in which each batch sized $z\, w_1$ to the retailer. The weight of each batch of inventory is:

$$zw_1 = \frac{x\, Y\, w_1\,(1-a)}{n} \tag{19}$$

The screening incurs a fixed cost for shipping batches of good quality to the retailer. Since the processor ships n batches during one period, the shipping cost per cycle (KC_s) is:

$$\mathrm{KC}^s = n\, K^s \tag{20}$$

The screening process is charged l per unit weight of the entire items. Therefore, the cost of screening process (LC_s) is:

$$\mathrm{LC}^s = l\, Y\, x\, w_1 \tag{21}$$

Total holding cost per cycle in the screening process (HC_s) is calculated by:

$$\mathrm{HC}^s = h^s \left(\frac{Y^2\, x^2\, w_1{}^2}{s} - \frac{(n-1)\, Y^2\, x^2\, w_1{}^2\,(1-a)}{2\, n\, s}\right) \tag{22}$$

The total cost in the screening process (TC_s) is calculated by adding up Equations (20), (21), and (22) as follow:

$$TC^s = n\, K^s + l\, Y\, x\, w_1 + h^s \left(\frac{Y^2\, x^2\, w_1{}^2}{s} - \frac{(n-1)\, Y^2\, x^2\, w_1{}^2\,(1-a)}{2\, n\, s}\right) \tag{23}$$

The processor sends all batch of good quality during one processing cycle with a combined weight of $Y\, x\, w_1(1 - a)$. The processor will receive a fee (P_p) from the retailer for each unit weight of a good quality item by offering an incremental discount. The processor's income (TR_p) can be calculated by the equation:

$$TR^p = P^p\, Y\, x\, w_1 \tag{24}$$

After the screening process, the processor sells the poor quality processed at a cost of P_q per weight to the secondary market in one batch. Therefore, the income from the sale of poor-quality products is:

$$TR^q = P^q\, Y\, x\, a\, w_1 \tag{25}$$

The total profit of the processor (TP_p) is the sum of the sales of the good quality and poor quality in equations (24) and (25) minus the costs incurred by the processors in equations (16) and (23).

$$\begin{aligned} TP^p &= P^p\, Y\, x\, w_1 + P^q Y\, x\, a\, w_1 - K^p \\ &-P^f\, Y\, x\, w_1 - h^p \left(\frac{Y^2\, x^2\, {w_1}^2}{2\,R}\right) - n\, K^s \\ &-l\, Y\, x\, w_1 - h^s \left(\frac{Y^2\, x^2\, {w_1}^2}{s} - \frac{(n-1)\, Y^2\, x^2\, {w_1}^2\, (1-a)}{2\,n\,s}\right) \end{aligned} \tag{26}$$

Retailer's Profit

Retailers need to meet the level of demand D from the final consumer. To meet this demand, the retailer receives n batch of size $z\, w_1$ from the processor. Overall, in one cycle period, the total weight sold by the retailer is $Y\, x\, w_1(1-a)$. Therefore, the total duration of successive order cycle times (T) is:

$$T = \frac{Y\, x\, w_1(1-a)}{D} \tag{27}$$

The retailer cycle order cost (KC_r)is:

$$KC^r = K^r \tag{28}$$

The retailer obtains good quality product y from the processor at an incremental discount price from the processor. So that the retailer's purchase cost from the processor (PC_r) is equal to the processor's sales, namely:

$$PC^r = TR^p = P^p\, Y\, x\, w_1 \tag{29}$$

The holding cost in retailer side (HC_r) can be calculated using the model adopted from Konstantaras *et al.* [12] as follow:

$$HC^r = h^r \left(\frac{Y\, x\, w_1(1-a)\, T}{2} - \frac{(n-1)\, Y^2\, x^2\, {w_1}^2\, (1-a)}{2\,n\,s}\right) \tag{30}$$

The retailer will sell good quality products to consumers by offering incremental discounts at a price P_r per unit weight. So that the retailer's revenue can be calculated by:

$$TR^r = R^r + P^r\, x\, w_1\, (1-a)\, (Y - Y_r) \tag{31}$$

The retailer's total profit (TP_r) is the total revenue of the retailer in Equation (31) minus the costs incurred by the retailer during one cycle period in Equations (28), (29), and (30).

$$\begin{aligned} TP^r &= R^r + P^r\, x\, w_1\, (1-a)\left(Y - Y_p\right) - K^r \\ &-P^p\, Y\, x\, w_1 - h^r \left(\frac{Y\, x\, w_1(1-a)\, T}{2} - \frac{(n-1)\, Y^2\, x^2\, {w_1}^2\, (1-a)}{2\,n\,s}\right) \end{aligned} \tag{32}$$

Maximization of Total Profit

The objective function of the model in this study is to maximize the total profit per unit time for the entire supply chain. Total supply chain profit is obtained from revenue minus total cost. The total cost includes several costs at each echelon, including purchase costs, setup costs, holding costs, screening costs, and mortality costs.

$$\begin{aligned} TP^{sc} &= \left[R^r + P^r\, x\, w_1\, (1-a)\left(Y - Y_p\right)\right] + P^q\, Y\, x\, a\, w_1 \\ &-K^r - h^r \left(\frac{Y\, x\, w_1(1-a)\, T}{2} - \frac{(n-1)\, Y^2\, x^2\, {w_1}^2\, (1-a)}{2\,n\,s}\right) - n\, K^s \\ &-l\, Y\, x\, w_1 - h^s \left(\frac{Y^2\, x^2\, {w_1}^2}{s} - \frac{(n-1)\, Y^2\, x^2\, {w_1}^2\, (1-a)}{2\,n\,s}\right) - K^p \\ &-h^p \left(\frac{Y^2\, x^2\, {w_1}^2}{2\,R}\right) - [R^v + P^v\, w_0\, (Y - Y_v)] \\ &-K^f - Y\left(c^f\, x + m^f\, (1-x)\right) \\ &\left\{\alpha\, T^f + \frac{\alpha}{\lambda}[\ln(1 + \beta e^{-\lambda\, T^f}) - \ln(1+\beta)]\right\} \end{aligned} \tag{33}$$

Total supply chain profit per cycle (TPU_{SC}) is obtained by dividing Equation (33) as the total supply chain profit by Equation (27) as the replenishment cycle time which resulting in the following equation:

$$\begin{aligned} TPU^{sc} &= \frac{D\, R^r}{(1-a)x\, Y\, w_1} + \frac{D\, P^r\, (Y - Y_r)}{Y} + \frac{a\, D\, P^q}{(1-a)} - \frac{D\, K^r}{(1-a)x\, Y\, w_1} \\ &-h^r \left(\frac{x\, Y\, w_1(1-a)}{2} - \frac{(n-1)x\, Y\, w_1 D}{2\,n\,s}\right) - \frac{D\, n\, K^s}{(1-a)x\, Y\, w_1} - \frac{D\, l}{(1-a)} \\ &-D\, h^s \left(\frac{x\, Y\, w_1}{s\,(1-a)} - \frac{(n-1)x\, Y\, w_1}{2\,n\,s}\right) - \frac{D\, K^p}{(1-a)x\, Y\, w_1} - \frac{D\, x\, Y\, h^p\, w_1}{2(1-a)R} \\ &-\frac{D\left(R^v + P^v\, w_0(Y - Y_v)\right)}{(1-a)xYw_1} - \frac{D\, K^f}{(1-a)xYw_1} \\ &-\frac{D}{(1-a)xw_1}\left(c^f\, x + m^f\, (1-x)\right)\left(\alpha\, T^f + \frac{\alpha}{\lambda}[\ln(1 + \right. \\ &\left. \beta e^{-\lambda\, T^f}) - \ln(1+\beta)]\right) \end{aligned} \tag{34}$$

The survival rate (x) and the poor quality product (a) are random variables which are assumed to be uniformly distributed with probability density functions of $f(x)$ and $f(a)$ respectively. So Equation (34) becomes:

$$\begin{aligned} E[TPU^{sc}] &= \frac{D\, R^r}{(1-E[a])\, E[x]Y\, w_1} + \frac{D\, P^r\, (Y - Y_r)}{Y} + \frac{E[a]D\, P^q}{(1-E[a])} \\ &-\frac{D\, K^r}{(1-E[a])\, E[x]Y\, w_1} - h^r \left(\frac{E[x]\, Y\, w_1(1-E[a])}{2} - \frac{(n-1)\, E[x]Y\, w_1 D}{2\,n\,s}\right) \\ &-\frac{D\, n\, K^s}{(1-E[a])\, E[x]Y\, w_1} - \frac{D\, l}{(1-E[a])} \\ &-D\, h^s \left(\frac{E[x]Y\, w_1}{s\,(1-E[a])} - \frac{(n-1)\, E[x]Y\, w_1}{2\,n\,s}\right) \\ &-\frac{D\, K^p}{(1-E[a])\, E[x]Y\, w_1} - \frac{D\;\; E[x]Y\, h^p\, w_1}{2(1-E[a])R} \\ &-\frac{D\left(R^v + P^v\, w_0(Y - Y_v)\right)}{(1-E[a])\, E[x]\, Y\, w_1} - \frac{DK^f}{(1-E[a])\, E[x]Y\, w_1} \\ &-\frac{D}{(1-E[a])\, E[x]w_1}\left(c^f\;\; E[x] + m^f\, (1 -\; E[x])\right) \\ &\left(\alpha\, T^f + \frac{\alpha}{\lambda}[\ln(1 + \beta e^{-\lambda\, T^f}) - \ln(1+\beta)]\right) \end{aligned} \tag{35}$$

The order quantity that maximizes the total supply chain profit can be determined by setting the first derivative of the objective function to zero to result as follow:

$$Y = \sqrt{\frac{2\,D\,(P^r\,Y_r\,(1-E[a])\,E[x]\,w_1 - R^r + K^r + n\,K^S + K_p + R^v - P^v\,w_0\,Y_v + K^f)}{(1-E[a])\,E[x]^{\,2}w_1^{\,2}\left(h^r\left((1-E[a])-\frac{(n-1)\,D}{n\,s}\right)+D\,h^s\left(\frac{2}{s\,(1-E[a])}-\frac{(n-1)}{n\,s}\right)+\frac{D\,h^p}{(1-E[a])R}\right)}} \tag{36}$$

Optimal Solution Search Algorithm

Searching for the optimal solution requires a procedure, in this proposed model a method is proposed direct search, which is a method by sequentially examining the trial solution and comparing each trial solution with the best value (Hooke & Jeeves, 1961). This research uses software Mathematica 11.2 to find a solution. The following algorithm is used to obtain a solution for the number of orders and the optimal cycle time.

Step 1 : Calculate R^v for each discount offered by the vendor using equation (3)
Step 2 : Calculate R^r for each discount offered by the retailer
Step 3 : Start with $n = 1$
Step 4 : Calculate Y for each purchase cost per unit using Equation (36)
Step 5 : Determine whether Y what is obtained is feasible or not at the farmer's purchase. Y declared feasible if Y is in the interval $Y_v \leq$ Y $< Y_{v+1}$. If the value Y is not feasible, then it is ignored.
Step 6 : Count T^f and T for each Y feasible using equations (7) and (27). Check whether $T^f \leq T$. If it meets, then go to step 7, if not, go to step 6b.
Step 6b : If $T^f \leq T$, equate the value T with T^f and use the new T value to determine the new value of Equation (27). Then go to step 8
Step 7 : Check whether $E[a] \leq 1 - \frac{D}{s}$. If it meets, then proceed to step 8, if not, then it cannot be continued.
Step 8 : Calculate $E[TPU^{sc}]$ using Eq. (35) for all Y eligible.
Step 9 : Increase the value n by 1, then repeat steps 4 to step 8. If the value increases, continue to step 10. If not, then the value $E[TPU^{sc}]$, Y and n the previous are the best solutions.
Step 10 : Finish.

Results and Discussions

Numerical Example

A numerical example is given to show the applicability of the model, and the parameters are mostly adapted from Sebatjane and Adetunji [11]. The following parameters are used in the numerical example: $D =$ 250 kg/week; $R = 300$ kg/week; $w_0 = 8.5$ kg; $w_1 = 30$ kg;$K^r = 2{,}500{,}000$ IDR; $h^r = 1{,}000$

Table 1. Farmer purchase prices with incremental discounts

Quantity purchase (unit)	Price (IDR/kg)
0 – 100	10,000
101 – 150	8,000
151 – 200	6,000
201+	4,000

Table 2. Retailer sales prices with incremental discounts

Quantity sale (unit)	Price (IDR/kg)
0 – 10	50,000
11 – 30	48,000
31 – 50	46,000
51+	44,000

IDR /kg/week; $K^p = 25{,}000{,}000$ IDR; $h^p = 500$ IDR kg/week; $K^S = 200{,}000$ IDR; $h^s = 500$ IDR / kg/week; $P^q = 20{,}000$ IDR/kg; $l = 500$ IDR /kg; $s = 1{,}000$ kg/week; $K^f = 30{,}000{,}000$ IDR; $c^f = 1{,}000$ IDR /kg/week; $m^f = 2{,}000$ IDR/kg/week; $\alpha = 51$ kg; $\beta = 5$; $\lambda = 0.12$ /week; x and a are assumed to be random variables uniformly distributed over [0.8, 1] and [0 , 0.05], respectively. Their probability density functions are given by:

$$f(x) = \begin{cases} 5, & 0.8 \leq x \leq 1 \\ 1, & \text{otherwise} \end{cases}$$

$$f(a) = \begin{cases} 32, & 0 \leq a \leq 0.05 \\ 1, & \text{otherwise} \end{cases}$$

This implies that

$$E[x] = \int_{0{,}8}^{1} 5x\,dx \;= 5\left[\frac{1^2-0.8^2}{2}\right] = 0.9$$

$$E[a] = \int_{0{,}8}^{1} 32a\,da \;= 32\left[\frac{0.05^2-0^2}{2}\right] = 0.04$$

New growing item vendor provides incremental discounts is summarized in Table 1, and the retailer sells the product by offering incremental discounts is summarized in Table 2.

Based on the algorithm, the optimal solution of the proposed model is shown in Table 3. The table shows the feasibility of quantity orders based on the given discount incremental range. For example, when $n = 1$ and the price of the newborn item are 10000 IDR, the quantity order is 158 items, but the range of that price is 0-100 items. Therefore this solution is infeasible. On the other hand, when $n = 1$ and the price of the newborn item are 6000 IDR, the quantity order is 161 items, and the range of that price is 151-200 items. Therefore this solution is feasible.

Based on the numerical example, the company should place an order of 182 units at the beginning of each cycle, with the number of delivery batches is nine times. Based on Eq. (27), the replenishment on

Table 3. Calculation results of numerical examples

n	Y_1	Y_2	Y_3	$E[TPU]$	Y_4
1	158	159	161*	728,629	162
2	168	170	171*	1,136,370	172
3	172	174	175*	1,271,150	176
4	174	176	177*	1,334,490	178
5	176	177	179*	1,368,730	180
6	177	179	180*	1,388,250	181
7	178	179	181*	1,399,290	182
8	178	180	182*	1,405,010	183
9	179	181	**182***	**1,407,210**	183
10	180	181	183*	1,406,910	184

*feasible

Table 4 Comparison between Sebatjane and Adetunji [11] and this research

Variables	Sebatjane and Adetunji(2020)	This Research
Y^*	179	182
n	9	9
T	18.57	18.86
TPU^*	2,177,290	1,407,210

the farmer's side is done every 18.86 weeks. Therefore, the total supply chain profit is IDR 1,407,210.

We perform a comparative analysis with the model of Sebatjane and Adetunji [11], and the results of such comparison are shown in Table 4. The table shows that the proposed model resulted in lower profits due to the imperfect quality not considered in the Sebatjane and Adetunji [11]. While for the decision variables, it only slightly differs between both types of research.

Sensitivity Analysis

Sensitivity analysis was carried out with scenarios of -50%, -25%, +25%, and +50% changes in the parameters value. The changes in the parameters value are listed in Table 5 and Table 6.

The effect of parameter changes on optimal order quantity, cycle time, total profit, and the number of batches can be seen in Figure 1.

Figure 1-3 shows that the decision variable for order quantity (Y) is sensitive to changes in farmer's setup cost (K^f), processing cost (K^p), processor's holding cost (h^p), retailer's holding cost (h^r), and item life expectancy ($E[x]$).

Figure 2 shows that the decision variable for cycle time (T) is sensitive to changes in the parameter of farmer's setup cost (K^f), processing cost (K^p), processor holding cost (h^p), and retailer's holding cost (h^r).

Figure 3 shows that the decision variable for the number of delivery batches (n) is sensitive to changes in the parameter of farmer's setup cost (K^f),

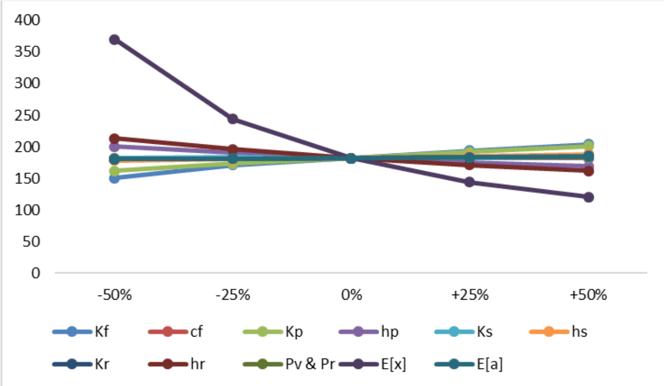

Figure 1. Effect of parameter changes on Y

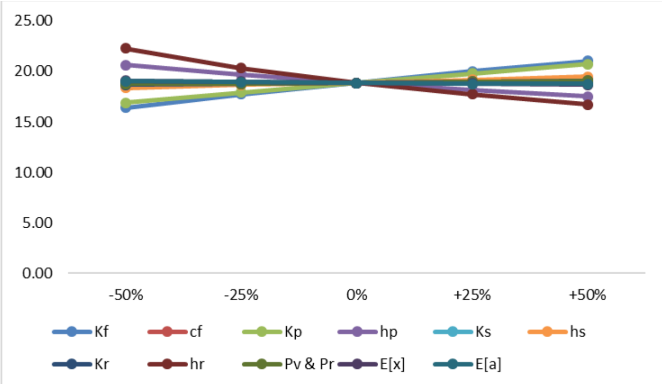

Figure 2. Effect of parameter changes on T

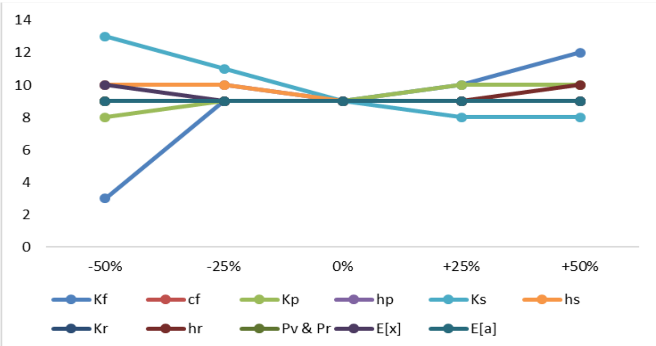

Figure 3 Effect of Parameter Changes on n

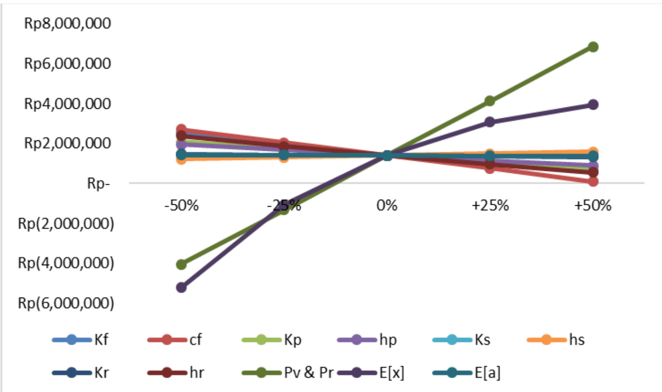

Figure 4 Effect of Parameter Changes on $E[TPU]$

processing cost (K^p), processor's holding cost (h^p), screening's setup cost (K^s), screening's holding cost (h^s), retailer's holding cost (h^r), item life expectancy ($E[x]$).

Table 5 Scenarios of changes in parameters value.

% Change	Parameters							
	K^f	c^f	K^p	h^p	K^s	h^s	K^r	h^r
-50%	15,000,000	500	12,500,000	250	100,000	250	1,250,000	500
-25%	22,500,000	750	18,750,000	375	150,000	375	1,875,000	750
0%	30,000,000	1.000	25,000,000	500	200,000	500	2,500,000	1000
25%	37,500,000	1.250	31,250,000	625	250,000	625	3,125,000	1250
50%	45,000,000	1.500	37,500,000	750	300,000	750	3,750,000	1500

Table 6 Scenarios of Changes in parameters value.

% Change	Parameters									
	P^v & P^r								$E[x]$	$E[a]$
	P_1^v	P_2^v	P_3^v	P_4^v	P_1^r	P_2^r	P_3^r	P_4^r		
-50%	5,000	4,000	3,000	2,000	25,000	24,000	23,000	22,000	0.450	0.02
-25%	7,500	6,000	4,500	3,000	37,500	36,000	34,500	33,000	0.675	0.03
0%	10,000	8,000	6,000	4,000	50,000	48,000	46,000	44,000	0.900	0.04
25%	12,500	10,000	7,500	5,000	62,500	60,000	57,500	55,000	1.125	0.05
50%	15,000	12,000	9,000	6,000	75,000	72,000	69,000	66,000	1.350	0.06

Figure 4 shows that the objective function for total profit per unit time ($E[TPU]$) is sensitive to changes in the parameters of farmer's setup cost (K^f), growing item feeding cost (c^f), processing cost (K^p), processor's holding cost (h^p), screening's holding cost (h^s), retailer's holding cost (h^r), item price, and item life expectancy ($E[x]$). From the results of the sensitivity analysis, managers can increase total profit by taking care of growing items, maintaining cleanliness, preparing medicine, providing adequate nutrition to increase the number of survival items which at the same time will decrease of holding cost and mortality cost for dead items.

Conclusion

This research developed an optimization model for determining the optimal order quantity for growing items by considering the imperfect quality and incremental discounts in a three-echelon supply chain. The objective function of the model is to maximize the total supply chain profit. The decision variables in this study included the optimal order quantity, the retailer's cycle time which is used as the primary cycle time, and the number of delivery batches from the processor to the retailer. The comparative analysis shows that the proposed model is different from the reference model. The proposed model resulted in a slightly higher number of ordered newborn items with lower supply chain profit. This indicated that incremental discounts have a considerable impact on inventory management. The sensitivity analysis results show that the decision variables and the objective functions are significantly affected by the probability of the live items surviving throughout the growth period. By taking care of and decreasing the number of dead items, managers can increase the total profit and decrease the order quantity and holding cost. For future research, it is necessary to consider further development, such as allowing several products (multi-item) and an optimal selling price.

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