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Optimal Retention for a Quota Share Reinsurance

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Abstract: The Indonesian Financial Services Authority (OJK) has instructed all insurance providers in Indonesia to apply a mandatory tariff for property insurance. The tariff has to be uniformly applied and the rule to set the maximum and minimum premium rates for protection against losses should be applied. Furthermore, the OJK issued the new rule regarding self-retention and domestic reinsurance. Insurance companies are obliged to have and implement self-retention for each risk in accordance with the self-retention limits. Fluctuations in total premium income and claims may lead the insurance company cannot fulfill the obligation to the insured, thus the company needs to conduct reinsurance. Reinsurance helps protect insurers against unforeseen or extraordinary losses by allowing them to spread their risks. Since reinsurer chargers premium to the insurance company, a properly calculated optimal retention would be nearly as high as the insurer financial ability. This paper is aimed at determining optimal retentions indicated by the risk measure Value at Risk (VaR), Expected Shortfall (ES) and Minimum Variance (MV). Here we use the expectation premium principle which minimizes individual risks based on their quota share reinsurance. Regarding the data in an insurance property, we use a bivariate lognormal (BLN) distribution to obtain VaR, ES, and MV, and a bivariate exponential (BEXP) distribution to obtain MV. The bivariate distributions are required to derive the conditional probability of the amount of claim occurs given the benefit has occurred. We find that first, based on fitting distribution, the use of bivariate lognormal distribution is suitable for the determination of retention and the value of MV is between VaR and ES. Second, the value of MV-BEXP is less than MV-BLN that have an impact on high reinsurance‘s value.

Keywords: Property quota share insurance optimal retention; minimum variance; Value at Risk; expected shortfall; bivariate lognormal distribution; bivariate exponential distribution.

Introduction

The insurer's risk can be reduced by the use of reinsurance, in which another insurer (the reinsurer) acts as an insurer of the insurance company (the ceding company) originally covering the risk (Panjer and Willmot [1]). Fluctuations in total premium income and the payment of claims may lead the insurance company cannot fulfill the obligation to the insured, thus the insurance company needs to conduct reinsurance. Reinsurance helps protect insurers against unforeseen or extraordinary losses by allowing them to spread their risks. Since reinsurer chargers premium to the insurance company, a properly calculated optimal retention would be nearly as high as the insurer financial ability. The insurance company has to set a retention limit which is the amount below which this company will retain the reinsurance and above which it will purchase reinsurance coverage from another (the reinsuring) company.

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Optimization proportional reinsurance problems have been considered by many researchers in recent year. Since the cornerstone work by Borch [2], the study of optimal reinsurance has drawn a significant interest from both actuaries and academics. Using variance of retained loss as the criterion and under the assumption that the reinsurance premium is determined by the expected premium principle, he showed that the stop loss reinsurance is optimal.

Cai, and Tan [3] proposed practical solutions for the determination of optimal retentions in a stop-loss reinsurance based on minimizing VaR and ES respectively, the VaR and the ES of the cedent's total risk exposure, based on quota share and stop-loss reinsurance. Pressacco, *et al.* [4] gave a closed-form formula to express the efficient mean-variance retention set both in the retention space and in the mean-variance one. Kaluszka [5] derived optimal reinsurance arrangements balancing the risk measured by variance and expected profits under various mean-variance premium principles of the reinsurer. A description of these studies can be found for example in Kaluszka [6]; Cai *et al.* [7]; Balbas *et al.* [8]; Tan *et al.* [9]. Soleh, A.Z., *et al.* [10] consider a compound Poisson-Lognormal distribution in determining the retention of stop-loss reinsurance.

Our approach exploits the risk measure based on Cai and Tan [3] and Tan *et al.* [9]. Previous studies only consider the amount of claims, without observing the amount of benefits, whereas Noviyanti, *et al.* [11] observedamount of benefits and presented optimal reinsurance using the expected shortfall. The retention determination studies described before will be considered by regulators in decision making.

Effective as at 1 February 2014, The Indonesian Financial Services Agency (Otoritas Jasa Keuangan/ OJK) has instructed all insurance providers in Indonesia to apply a mandatory tariff for property insurance. The draft was first circulated publicly on Dec. 24, 2013, and implemented on Feb. 1, 2014, for all new policies and renewal policies with the period of insurance starting from Feb. 1, 2014, irrespective of the date of issue. Furthermore, based on Surat Edaran OJK Nomor 31/SEOJK.05/2015 Tahun 2015, the OJK issued the new rule regarding self-retention and domestic reinsurance. Insurance companies are obliged to have and implement self-retention for each risk in accordance with the self-retention limits. Insurance companies must submit their first reinsurance support strategy to OJK and must also develop and implement a reinsurance support strategy to ensure that they have sufficient capacity to meet their liabilities. Accordingly, this paper is aimed at determining optimal retentions indicated by VaR, ES, and MV.

VaR has become the generally accepted risk measure for financial risk management. It is consistent with the way that many insurances and other financial institutions currently measure and manage their risk. The European Commission, the Committee of European Insurance and Occupational Pensions Supervisors are carrying out a fundamental review of the regulatory capital regime for insurance companies, known as Solvency II, with the aim of establishing an improved solvency system that protects the interests of policyholders by reducing the probability of ruin, assuming a VaR measure. Despite its global acceptance, VaR has been criticized on certain theoretical grounds. It is worth pointing out that the International Actuarial Association Solvency Working Party appears to favor ES as a suitable risk measure in insurance (Noviyanti [12]). Whereas MV is related to the Markowitz methodology of optimal portfolio, i.e. the cedent is interested in minimizing the variance of his retained risk. Here we use the expectation premium principle which minimizes individual risks based on their quota share reinsurance. Regarding the data in an insurance property, we use a bivariate lognormal distribution to obtain VaR, ES and MV, and a bivariate exponential distribution to obtain MV. The bivariate distributions are required to derive the conditional probability of the amount of claim occurs given the benefit has occurred. This study will be useful for determining the fair value of retention of insurance companies. It should be noted that OJK sets the self-retention on general insurance companies (conventional and sharia insurance) separately. Data used in this study are data self-retention in the general insurance company, which is claim due to fire on a property in the form of a house. Therefore the final result obtained from this study is the value of retention partially.

Methods

Quota Share Reinsurance

Quota share reinsurance constitutes a type of pro rata or proportional in which the primary insurer and the reinsurer share the amounts of insurance, policy premiums and losses using a fixed percentage. All liability and premiums are shared from the first dollar. "Quota" or "definite" share relates to the fixed percentage as stated in the treaty. On premiums ceded, the reinsurer pays the ceding company a commission. The commission to the ceding company is an important factor in quota share reinsurance as it provides a financial benefit to the primary company. Proportional forms are often used in property insurance since this form provides catastrophic protection in addition to individual risk capacity (Kaufhold and Lennartz [13]).

Define C_1 as benefit random variable and C_2 as the amount of claim or claim severity random variable. Benefit constitutes a payment, such as one made under an insurance policy. We define conditional random variable of C_2 given C_1 or $X = (C_2|C_1 = c_1)$. We assume that X is a non-negative random variable with distribution function $FX(x) = Pr\{X \le x\}$, survival function $S_X(x) = Pr\{X > x\}$, and mean $E[X] > 0$. Let X_I and X_R be respectively, the reinsured amount to be covered by the insurance company and the amount that the insurance company retains it self. The relation of X_I and X_R with X and $r \in [0,1]$ is

$$X_I = rX \text{ and } X_R = (1-r)X \tag{1}$$

The cedent transfers risk by retaining $(1-r)$ proportion of the aggregate loss, and the reinsurer is liable for the remaining r proportion. Note that $r = 0$ denotes that the insurer retained all losses and $r = 1$ represents the insurer transferring all losses to a reinsurer.

A number of premium principles have been proposed for determining the appropriate level at the premium. One of the commonly used principles is the expected value principle in which the reinsurance premium is determined by

$$\delta(R) = (1+\rho)E(X_R) \tag{2}$$

The notation of $E[X_R]$ constitutes the net quota share premium. The total cost of the insurer in the presence of the quota share reinsurance T is captured by two components: the retained loss and the reinsurance premium, i.e.,

$$\begin{aligned} T &= X_I + \delta(R) \\ &= X_I + (1+\rho)E(X_R) \end{aligned} \quad (3)$$

Optimal Retention (VaR - ES Optimization)

VaR is more appropriately referred to the quantile risk measure since $VaR_\alpha(X)$ is exactly a (1-α)-quantile of the random variable X. It follows from the definition of $VaR_\alpha(X)$ that

$$VaR_\alpha(X) \le x \Leftrightarrow S_X(x) \le \alpha \quad (4)$$

Define $\rho^* = (1+\rho)^{-1}$, where $\rho > 0$ is known as the relative safety loading, for example, add expenses percentage, operational and administration fees. The survival function of the retained loss X is given by

$$S_x(\rho^*) = 1 - P_r\left((C_2|C_1 = c_1) \le S_x^{-1}(\rho^*)\right) = \rho^* \quad (5)$$

Referring to (6), this should be the conditional probability of C_2 occurs given C_1 has occurred, is represented as

$$P_r((C_2|C_1 = c_1) \le S_x^{-1}(\rho^*)) = 1 - \rho^* \quad (6)$$

If $X = (C_2|C_1 = c_1)$ and using the central limit theorem,

$$z = \frac{(C_2|C_1=c_1) - E(C_2|C_1=c_1)}{\sqrt{V(C_2|C_1=c_1)}} \to \sim N(0,1), \quad (7)$$

and

$$P_r\left(z \le \frac{S_x^{-1}(\rho^*) - E(C_2|C_1=c_1)}{\sqrt{V(C_2|C_1=c_1)}}\right) = 1 - \rho^*, \quad (8)$$

Then,

$$S_x^{-1}(\rho^*) = E(C_2|C_1 = c_1) + z_{(1-\rho^*)}\sqrt{V(C_2|C_1 = c_1)} \quad (9)$$

The optimal retention $R^* > 0$ that minimizes $VaR_\alpha(X)$ exists if and only if both $\alpha < \rho^* < S_X(0)$ and

$$S_X^{-1}(\alpha) \ge S_X^{-1}(\rho^*) + \delta(S_X^{-1}(\rho^*)) \quad (10)$$

are held. When the optimal retention R^* exists, then $R^*$is given by

$$R^* = S_X^{-1}(\rho^*)$$

A prudent risk management is to ensure that risk measures associated with T are as small as possible. This information motivates us to consider the following two optimization criteria for seeking the optimal level of retention. First, if the measurement of the risk used is VaR:

$$VaR_T(R,\alpha) = VaR_{XI}(R,\alpha) + \delta(R) \quad (11)$$

The VaR based on Tan *et al.* [9] optimization is defined as:

$$VaR_T(R^*,\alpha) = min_{R>0}\{VaR_{XI}(R,\alpha)\} \quad (12)$$

and it will be minimum given by

$$\begin{aligned} R^*_{VaR(\rho^*)} &= S_x^{-1}(\rho^*)(1-r) \\ &= S_x^{-1}(1/1+\rho)(1-r) \end{aligned} \quad (13)$$

where $\alpha \le \rho^* \le S_x(o); \rho^* = 1/1+\rho$

Second, if the measurement of the risk used is ES:

$$ES_T(R,\alpha) = ES_\alpha(R,r) + \delta(R) \quad (14)$$

The ES based on Tan *et al.* [9] optimization is defined as:

$$ES_T(R^*,\alpha) = \underset{R>0}{min}\left(ES_\alpha(R,r)\right) \quad (15)$$

and it will be minimum given by

$$R^*_{ES(\rho^*)} = (1-r)S_x^{-1}(\rho^*) + \frac{1}{\rho^*}\int_{S_x^{-1}(\rho^*)}^{\infty} S_x(x)dx \quad (16)$$

The relationship between the VaR and the ES is as follow,

$$ES_T(R^*,\alpha) = E[T|T \ge VaR_T(R^*,\alpha)] \quad (17)$$

The resulting optimal retention R^* ensures that both the VaR and the ES of the total cost is minimized for a given risk tolerance level α.

The survival function of retained loss X_I is given by

$$S_{X_I}(x) = \begin{cases} S_X(x), 0 \le x \le R, \\ 0, x > R, \end{cases} \quad (18)$$

and the VaR and the ES of the retained loss X_I can be represented as

$$VaR_{X_I}(R,\alpha) = \begin{cases} R & ,0 < R \le S_X^{-1}(\alpha), \\ S_X^{-1}(\alpha) & ,R > S_X^{-1}(\alpha). \end{cases} \quad (19)$$

$$ES_\alpha(X_i,r) = (1-r)S_x^{-1}(\alpha) + \frac{1}{\alpha}\int_{S_x^{-1}(\alpha)}^{\infty} S_X(x)\,dx \quad (20)$$

A simple relationship between the VaR-ES of the total cost and the VaR-ES of retained risk respectively are

$$VaR_T(R,\alpha) = VaR_{X_I}(R,\alpha) + \delta(R) \quad (21)$$
$$ES_T(R,\alpha) = ES_\alpha(R,r) + \delta(R) \quad (22)$$

By combining (7) and (8), Tan *et al.* [9] summarize in the following proposition:

Proposition 1. *For each* $R > 0$ *and* $0 < \alpha < Sx(0)$, *then*

$$VaR_T(R,\alpha) = \begin{cases} R + \delta(R), & 0 < R \le S_X^{-1}(\alpha), \\ S_X^{-1}(\alpha) + \delta(R), & R > S_X^{-1}(\alpha). \end{cases} \quad (23)$$

$$ES_T(X_I,\alpha) = (1-r)S_x^{-1}(\alpha) + \delta(R) + \frac{1}{\alpha}\int_{S_x^{-1}(\alpha)}^{\infty} S_X(x)\,dx. \quad (24)$$

Let define$\rho^* = 1/(1+\rho)$ which plays a critical role in the solutions to our optimization problems. The theorem states the necessary and sufficient conditions for the existence of the optimal retention of the VaR-ES optimization:

Theorem 1.

a) *If both*

$$\alpha < \rho^* < S_X(0), \tag{25}$$

and

$$S_X^{-1}(\alpha) \geq S_X^{-1}(\rho^*) + \delta\left(S_X^{-1}(\rho^*)\right) \tag{26}$$

are fulfilled, then the optimal retention R^*>0 exists.

b) *When the optimal retention R^* in* (4) *exists, then R^* is given by*

$$R^* = S_X^{-1}(\rho^*) \tag{27}$$

and the minimum VaR of T is given by

$$VaR_T(R^*, \alpha) = R^* + \delta(R^*) \tag{28}$$

and the minimum ES of T is given by

$$ES_T(R^*, \alpha) = R^* + \delta(R^*) \tag{29}$$

Corollary 1. *If both* (10) *and* (11) *are fulfilled then*

$$S_X^{-1}(\alpha) \geq (1+\rho)E[X], \tag{30}$$

the optimal retention R^*>0 exists and the minimum ES can be formulated as (12) *and* (14)*, respectively.*

VaR and ES Based on Bivariate Log Normal Distribution

Let $C_1 \sim LN(\mu_1, \sigma_1)$, $C_2 \sim LN(\mu_2, \sigma_2)$, and $(c_1, c_2) \sim LN$ *Bivariate* $\left((\mu_1,\mu_2),(\sigma_1^2,\sigma_2^2,\rho_{1,2})\right)$ then the probability density function of LN Bivariate is given by

$$f(c_1, c_2) = \frac{1}{2\prod \sigma_1\sigma_2\sqrt{1-\rho_{1,2}^2}c_1c_2} e^{-\frac{1}{2}w} \tag{31}$$

where $-1 \leq \rho_{1,2} \leq 1$ and

$$w = \frac{1}{1-\rho_{1,2}^2}\left\{\left(\frac{\ln c_1 - \mu_1}{\sigma_1}\right)^2 - 2\rho_{1,2}\left(\frac{\ln c_1-\mu_1}{\sigma_1}\right)\left(\frac{\ln c_2-\mu_2}{\sigma_2}\right) + \left(\frac{\ln c_2-\mu_2}{\sigma_2}\right)^2\right\} \tag{32}$$

Since $X = (C_2|C_1 = c_1)$ then

$$E(C_2|C_1 = c_1) = (c_1)^{\left(\sigma_2/\sigma_1\right)\cdot\rho_{12}} \cdot exp\left(\mu_2 - \frac{\sigma_2}{\sigma_1}\cdot\rho_{12}\cdot\mu_1 + \frac{1}{2}\sigma_2^2(1-\rho_{12}^2)\right) \tag{33}$$

and

$$V(C_2|C_1 = c_1) = (c_1)^{2\left(\sigma_2/\sigma_1\right)\cdot\rho_{12}} \cdot exp\left(2\left(\mu_2 - \frac{\sigma_2}{\sigma_1}\cdot\rho_{12}\cdot\mu_1\right)\right)\cdot exp(z)\cdot(exp(z)-1) \tag{34}$$

where $z = \sigma_2^2(1-\rho_{12}^2)$ and $\rho_{12} = \frac{cov(C_1,C_2)}{\sigma_1\sigma_2}$.

Substituting (12) and (13) into (9) respectively we obtain the optimal retention (28) for VaR and (29) for ES.

Minimum Variance (MV) based on Bivariate Log Normal and Bivariate Exponential Distribution

The risk measurement Minimum Variance based on Kaluszka [5] is defined as:

$$Minimum\ Var\left[\sum_{i=1}^{N}\left(x_i - R(x_i)\right)\right] \tag{35}$$

subject to

$$D\left(\sum_{i=1}^{N} R(x_i)\right) \leq g\left(p, E\sum_{i=1}^{N} R(x_i)\right),$$

and

$$E\left[\sum_{i=1}^{N}\left(x_i - R(x_i)\right)\right] = mE[N], 0 \leq R \leq x.$$

The optimal retention using Minimum Variance is as follows:

$$R_{MV}^*(\rho^*) = \frac{(1-r\lambda)}{\rho^*}E(C_2|C_1 = c_1)\rho^* = \frac{1}{1+\rho} \tag{36}$$

If$(C_2|C_1 = c_1) \sim LN\ Bivariate\ \left((\mu_1,\mu_2),(\sigma_1^2,\sigma_2^2,\rho_{12})\right)$. for $\lambda = 1$, the value of optimal return is:

$$R_{MVLNB}^*(\rho^*) = \frac{(1-r\lambda)}{\rho^*}E(C_2|C_1 = c_1)\rho^* = \frac{1}{1+\rho}$$
$$= (1-r)\left\{(C_1)^{\hat{\sigma}_2/\hat{\sigma}_1\rho_{12}}\cdot exp\left(\mu_2 - \frac{\sigma_2}{\sigma_1}\rho_{12}\mu_1 + \frac{1}{2}\sigma_2^2(1-\rho_{12})^2\right)\right\}(1+\rho).$$

If $(C_2|C_1 = c_1) \sim Exp\ bivariate\ (\delta)$

$$f(C_2|C_1) = e^{-C_2(1+\hat{\delta}C_1)}\left[\left(1+\hat{\delta}C_2\right)\left(1+\hat{\delta}C_1\right) - \delta\right] \tag{38}$$

The conditional expectation is

$$E(C_2|C_1 = c_1) = \frac{1+\hat{\delta}+\hat{\delta}C_1}{\left(1+\hat{\delta}C_1\right)^2}E(C_2) \tag{39}$$

for $\lambda = 1$, and the value of optimal return is:

$$R_{MVEXPB}^*(\rho^*) = \frac{(1-r)}{\rho^*}E(C_2|C_1 = c_1)$$
$$= (1+\rho)(1-r)\left(\frac{1+\hat{\delta}+\hat{\delta}C_1}{\left(1+\hat{\delta}C_1\right)^2}\right)E(C_2) \tag{40}$$

Exponential distribution is a good reference distribution for example, but usually too "optimistic" for a real model. The Exponential distribution is often used to describe the inter-arrival time of loss events. For an exponentially distribution random variable$x$, we write $x \sim \exp(\lambda)$, where $\lambda > 0$ is a parameter.

Results and Discussions

In this section, we illustrate how to determine retention value. Let C_1 and C_2 represent the benefit and the amount of claim respectively. We have to determine the conditional probability that C_2 occurs given C_1 has occurred. If $(C_1, C_2) \sim$LogNormal Bivariate $((\mu_1, \mu_2), (\sigma_I^2, \sigma_2^2, \rho))$ then the bivariate density function, expectation, and variance are listed in equation (31), (33) and (34), respectively.

We deeply investigated the consequences on optimal retention of a combination of loading factor (ρ=20%) and different quota share proportion (r = 10%, 20%, 30%, and 40%). These values are based on the experience of the insurance company, OJK does not determine both loading factor and quota share proportion but rather to determine the value of retention. According to Table 1, we have the expectation and the variance of the amount of claim distribution using the Bivariate Log Normal distribution.

To find the value of $S_X^{-1}(\rho)$; $0 \leq \rho \leq 1$, and based on the central limit theorem, the distribution of the number of claims can be approximated using a normal distribution. In order to demonstrate an application of our theoretical derivation, we use an example from the occurrence of a natural disaster i.e. fire insurance. Suppose $\alpha = 0.1$, the weight $r = 0.1$ and the loading premium $\rho = 0.1$. These values are based on the experience of the insurance company, represented a good number. First, it will be checked in advance about the existence of retention.

$$\rho^* = \left(\frac{1}{1+\rho}\right) = \left(\frac{1}{1+0.2}\right) = 0.8333$$

Since the value of ρ^* is 0.8333 i.e. between 0.1 and 1, then the necessary condition is fulfilled. If benefit IDR 1.000.000.000 and using equation (6), it will be calculated the retention, multiplied by the quota share.

$$\begin{aligned} S_X(0.8333) &= P(X > x) \\ x &= S_X^{-1}(0.8333) = 106{,}984{,}813.18. \end{aligned}$$

The optimal retention for VaR based on a Bivariate Log Normal distribution is the following:

$$\begin{aligned} R^*_{VaRLNB(\rho^*)} &= S_X^{-1}(0.8333)\,.\,(1-0.1) \\ R^*_{VaRLNB(\rho^*)} &= 106{,}984{,}813.18(1-0.1) \\ &= \text{IDR } 96.286.332 \end{aligned}$$

By using VaR as a risk measure, IDR 96.286.332 represented the amount of money the insurance company should incur in case of a claim form the certain policyholder.

The optimal retention for ES based on a Bivariate Log Normal distribution is:

$$R^*_{ESLNB(0.8333)} = (1-0.1)S_X^{-1}(0{,}8333) + \frac{1}{0.8333}\int_{106{,}984{,}813.18}^{\infty} S_X(x)dx = \text{IDR} \quad 327.884.200$$

The optimal retention for MV base on a Bivariate Log Normal distribution is:

$$R^*_{MVLNB}(0.8333) = \frac{(1-0.1\lambda)}{0.8333}E(C_2|C_1 = 1.000.000.000)$$

$$= (1-0.1) \left\{ \begin{matrix} (1{,}000{,}000{,}000)^{(0.7706/1.474).(0.578)}.\exp(19.1833 - (0.7706/1.474)) \\ (0.578)(22.078) + \frac{1}{2}(0.7706)^2(1-(0.578)^2 \end{matrix} \right\} (1+0.2) = \text{IDR} \quad 186.289.651$$

The optimal retention for MV base on a Bivariate Exponential distribution is:

$$R^*_{MVEXPB}(0.8333) = \frac{(1-0.1)}{0.8333}E(C_2|C_1 = 1.000.000.000)$$

$$= (1+0.2)(1-0.1)$$

$$\left(\frac{1+3.433.E-9+3.433.E-9.(1.000.000.000)}{\left(1+3.433.E-9.(1.000.000.000)\right)^2}\right)(291.239.515{,}4)$$

$$= \text{IDR } 88.745.273$$

The result of fitting distribution show that p-value of log-normal distribution is 0.8483 and p-value of exponential distribution is 0.1448 so that we can conclude that log normal distribution is more appropriate for the data than exponential distribution

Table 2 shows the amount of retention and reinsurance using the various scenarios. Based on Table 1, we have the expectation and the variance of the amount of claim distribution.

The loading factor, $\rho = 20\%$, is the percentage of additional expenses, operational and administration fees on the insurance company. Related to the fitting distribution, by setting the value of benefit as IDR 1 million (C_1), and the claim amount estimation as IDR 364,315,778 (C_2), the loss absorbed by the reinsurer is 10% (r) and the loss retained by the cedent is 90% ($1-r$), the optimal retention based on VaRLNB, ESLNB and MVLMB respectively are IDR 96,286,332, IDR 327.884.200 and IDR 186.289.651. Based on Table 2, each the r value can be arranged as $R^*_{VaRLNB} < R^*_{MVLNB} < R^*_{ESLNB}$. Whereas the value of reinsurance is the difference between benefit and retention. The greater loss retained by the cedent $(1-r)$, the smaller retention indicating that insurance companies are increasingly pessimistic.

Related to the risk MV measurement, the optimal retention of MVLNB and MVEXPB have the following relation $R^*_{MVEXPB} < R^*_{MVLNB}$

It should be noted that OJK sets the self retention on general insurance companies (conventional and sharia insurance) separately. Data used in this study are data self-retention in the general insurance company, which is claim due to fire on a property in the form of a house. Therefore the final result obtined from this study is the value of retention partially.

Table1. Estimation of benefit and amount of claim distributions (the Bivariate Log Normal (BLN) Distribution)

Benefit (C_1)	Amount of claim (C_2)
$C_1 \sim Lognormal(\mu_1, \sigma_1{}^2)$	$C_2 \sim Lognormal(\mu_2, \sigma_2{}^2)$
$f_c(C_1) = \frac{1}{1.474\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln C_1 - 22.078}{1.474}\right)^2\right)$	$f_c(C_2) = \frac{1}{.7706\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln C_2 - 19.1833}{.7706}\right)^2\right)$
$\hat{\mu}_1 = 22.078$	$\hat{\mu}_2 = 19.1833$
$\hat{\sigma}_1 = 1.474$	$\hat{\sigma}_2 = 0.7706$

Table 2. Estimation of amount of claim and retention (benefit = IDR 1,000,000,000,- and ρ = 20%)

	r	Claim amount estimation (IDR)	Retention (IDR)	Reinsurance (IDR)
VaRLNB	10%	364,315,778	96,286,332	903,713,668
	20%		85,587,851	914,412,149
	30%		74,889,369	925,110,631
	40%		64,190,888	935,809,112
ESLNB	10%		327,884,200	672,115,800
	20%		291,452,622	708,547,378
	30%		255,021,045	744,978,955
	40%		218,589,467	781,410,533
MVLNB	10%		186,289,651	813,710,349
	20%		165,590,801	834,409,199
	30%		144,891,950	855,108,050
	40%		124,193,100	875,806,900
MVEXPB	10%		88,745,273	911,254,727
	20%		78,884,687	921,115,313
	30%		69,024,101	930,975,899
	40%		59.163.515	940.836.485

Conclusion

In this paper, we keep as a starting point with the result concerning the risk measure VaR and ES from Cai and Tan [3] and Tan *et al.* [9], and the risk measure MV from Kaluszka [5]. Moreover, we elaborated these risk measures i.e. optimal retentions indicated by the risk measure Value at Risk (VaR), Expected Shortfall (ES) and Minimum Variance (MV) to individual risk model with dependent risk. Here we use the expectation premium principle which minimizes individual risks based on their quota share reinsurance. Regarding the data in an insurance property, we use a bivariate lognormal distribution to obtain VaR, ES and MV, and a bivariate exponential distribution to obtain MV. The bivariate distributions are required to derive the conditional probability of the amount of claim occurs given the benefit has occurred. The result of this study shows that bivariate log normal distribution take effect on the value of retention. However, the value of self-retained was established by OJK for all of the line business in the insurance company. Therefore, the value of R^* showed by this study cannot be compared yet with OJK policy. In future research, we will extend the work using other distributions and excess of loss reinsurance arranged on a claim by claim basis.

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