

Model of Dynamic Pricing for Two Parallels Flights with Multiple Fare Classes Based on Passenger Choice Behavior

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Abstract: Airline revenue management (ARM) is one of emerging topics in transportation logistics areas. This paper discusses a problem in ARM which is dynamic pricing for two parallel flights owned by the same airline. We extended the existing model on Joint Pricing Model for Parallel Flights under passenger choice behavior in the literature. We generalized the model to consider multiple full-fare class instead of only single full-fare class. Consequently, we have to define the seat allocation for each fare class beforehand. We have combined the joint pricing model and the model of nested Expected Marginal Seat Revenue (EMSR) model. To solve this hybrid model, we have developed a dynamic programming-based algorithm. We also have conducted numerical experiments to show the behavior of our model. Our experiment results have showed that the expected revenue of both flights significantly induced by the proportion of the time flexible passengers and the number of allocated seat in each full-fare class. As managerial insights, our model has proved that there is a closed relationship between demand management, which is represented by the price of each fare class, and total expected revenue considering the passenger choice behavior.

Keywords: Airline revenue management, parallel flights, dynamic pricing, passenger choice behavior, seat allocation.

Introduction

Airline industry is one of service industry that applies revenue management. It has limited capacity and time of product service offerings. Specifically, tickets offered by airlines have characteristics such as perishable products. They have no residual value if passed a certain period. That is, if the tickets unsold and the aircraft have to departure then revenue from unsold available seats will be lost.

Airline revenue management (ARM) is one of emerging topics in transportation logistics areas. The objective of ARM is to maximize passenger revenue by selling the right seats to the right customer at the right time (Dunleavy [4]). Airlines need to make decisions on “when” and “how many” ticket sold in high or low price to maximize the revenue.

In the literature, the models of ARM generally can be classified into two categories, static and dynamic models. The objective of the static model is to determine the number of seats that can be sold for each fare class.

In other the number of seats that can be sold for each fare class. In other words, to limit the availability of seats in different booking classes or “*booking limits*”. In this model, the decision has been made before the selling process begins and it will not be adjusted during the entire booking period. A prominent literature discussing static models is Belobaba [2]. This approach is known Expected Marginal Seat Revenue (EMSR). Some other papers are Smith *et al.* [7] and Weatherford and Bodily [10].

In the dynamic models, the booking control policy is not determined at the beginning of the booking period. The inventory seat allocation control should be dynamically reviewed over the entire booking period in order to optimize the expected revenue. Some papers discuss such problems including Lee and Hersh [6] and Subramanian, *et al.* [8] for discrete-time booking period and Feng and Xiao [5] for continuous-time booking model.

In more advanced models, some papers discuss the situation where an airline may open more than one flight in a day for “busy” routes. The airline opens some flights in the same departure date with different time schedule. These joint flights are called *multiple parallel flights*. The objective is to maximize the total revenue of the multiple parallel flights. In the multiple parallel flights, the airline should know the behavior of passengers in buying tickets. Some passengers may choose airlines not only based on

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price but also some other preferences such as schedules, aircraft types and in-flight service. In the point of view passengers, the seat availability of multiple parallel flights can be substituted each other. To face this situation, the airline should manage demand well. When making adjustment ticket fares over the booking period, the airline should consider the choice behavior of passengers among different flights.

In general, based on their preference, there are two types of passengers; those travel for either business purposes or leisure purposes. The first group usually has strong time preferences. They tend to value booking or cancellation flexibility. They are relatively price-insensitive since, in most cases, their travel expenses are not by themselves. On the other hand, the second group, leisure passengers, tends to be more sensitive to price since they pay the tickets by their own budget (Tallury and van Ryzin, [9]).

Specifically, based on their choice behavior, some researchers have categorized passengers into some groups. Zang and Cooper [12] have classified them into two groups, high fare and low fare passengers. Moreover, in the context of two parallel flights (e.g. flight A and flight B), Xiao *et al.* [11] classified potential passengers into three types. Passengers' type 1 and 2 are time-sensitive passengers. They buy only the ticket that is consistence with their departure-time requirements which are either flight A or flight B, respectively. They will never turn to buy the other flight even if lower price offered. In reality, most business travelers belong to this type. Passenger type 3 is price-sensitive passengers, which prefer to choose lower ticket price and do not consider the departure time. In reality, most leisure passengers belong to this type.

There are some papers discussing joint pricing for two parallel flights owned by the same airline. Zang and Cooper [12] proposed model in dynamic seat allocation control considering passenger choice behavior. Xiao *et al.* [11] also proposed dynamic pricing model for parallel flights problem. Moreover, Chen *et al.* [3] proposed model dynamic programming to optimize booking policy under seat allocation problem for two and multiple flight. However, none of these papers uses EMSR (Expected Marginal Seat Revenue) of Belobaba [2] which is commonly used in airline industry. We attempt to fulfill this research gap.

In our proposed model, we adopted EMSR approach to determine the seat allocation and booking level for each fare class in the cases of two parallel flights seat for a single-leg problem. We enhanced the EMSR model of Belobaba [2] to combine with the joint

dynamic pricing for parallel flights model of Xiao *et al.* [11]. The objective is to maximize total expected revenue of both flights by optimize seat allocation in each fare class.

First, we classified passengers into three types based on their choice behavior as of Xiao *et al.* [11]. We then determined seat allocation and booking level in each fare class using EMSR model. We finally applied dynamic programming to maximize the expected revenue of both flights.

Methods

Basic Models

Belobaba [2] developed nested EMSR approach model. This model is used to protect allocation seats of the higher fare level from the lower fare level. Using historical data, airlines can define demand level of each full-fare class. It is assumed that they have normal distribution with certain mean and standard deviation.

We adopted nested EMSR model to determine the seats allocation and booking level for each full-fare class. After defining the booking limits for each full-fare class, we then modified the model of Xiao *et al.* [11] on joint dynamic pricing to calculate the maximize expected total revenue of both flights. These two basic models will be explained in the next sub sections.

Seat Allocation Control Model of Belobaba [2]

There are two approaches to seat allocation problem, nested and non-nested allocations. Bazargan [1] has explained that in the nested allocation model, each fare class is assigned a booking limit. The booking limits are the total number of seats assigned to that fare class plus the sum of all seat allocations to its lower fare classes.

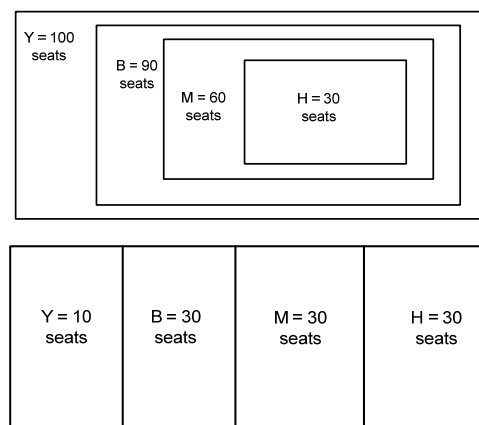


Figure 1. Nested and non nested seat allocations

In non-nested model, distinct numbers of seats are exclusively assigned to each fare class. The sum of this allocation adds up to the total aircraft seat capacity. Figure 1 shows the visualization of these two approaches.

The EMSR method of Belobaba [2] generates the nested protection level for different class fare. This method proposed that in a nested seat allocation, the number of seats which should be protected for higher fare class i , over class j (lower class) is:

$$EMSR(S_j^i) = f_i \cdot \bar{P}_i(S_j^i) = f_j \quad (1)$$

The protected number of seats for the $(n-1)$ fare class is determined by:

$$\Pi_{n-1} = \sum_{i=1}^{n-1} S_n^i \quad (2)$$

The average fare levels for the two classes of i and j are denote by f_i and f_j . S_j^i is the number of seats that should be protected for higher class i over class j , thus $\bar{P}_i(S_j^i)$ is the probability of selling S or more seats in fare class i .

The booking limit or the number of seat available for each class i , represented by BL_i , is determined by subtracting the number of seats protected for the higher fare class, Π_{i-1} , from the total aircraft seat capacity, C . Therefore:

$$BL_i = C - \Pi_{i-1} \quad (3)$$

With the booking limit for the highest fare class is:

$$BL_1 = C \quad (4)$$

The nested protection for fare class i is therefore the different between the booking limits for that fare and its lower fare class as follows:

$$NP_i = BL_i - BL_{i+1} \quad (5)$$

Where NP_i is the nested seat protection levels for fare class i .

Joint Dynamic Pricing by Xiao, et al. [11]

In this section we will elaborate the model of Xiao et al. [11] as the main reference of our model. This model considers a situation where there are only two flights (flight A and B) owned by a same airline and scheduled at different times. The objective is to maximize the total revenue of both flights during the booking period $[0, T]$.

As a nature of dynamic pricing, the ticket can be sold at k different prices. P is denoted as feasible price, in

which $P = \{p_1, p_2, \dots, p_k\}$ where $p_1 > p_2 > \dots > p_k$. p_1 is a full price whereas p_k is the lowest possible fare.

There are three types of passengers called Type I, Type II, Type III passengers, respectively. Type I (Type II) passengers select only flight A (or B), because they are time-sensitive passengers. Meanwhile Type III passengers will choose the flight with a lower price.

The fractions of three passenger types are $\alpha_i \in [0, 1]$, $i = 1, 2, 3$, $\sum \alpha_i = 1$. The potential passenger arrival process follows Poisson with rate λ . Let $\lambda_i = \alpha_i \cdot \lambda$, ($i = 1, 2, 3$) denotes the respective arrival rates of three types of passengers.

The authors assumed that the choice behavior of passengers is only applied when the perceive value of passengers towards to the ticket is greater than the selling price. If the selling prices of both flights are higher than the perceive value, the passenger will not buy the ticket.

When the potential passenger arrived, they make a choice based on the current offered price by flight A and B (p_A, p_B). If passenger belong to type I or II, then they will choose flight A or B if $X \geq p_A$, ($X < p_B$). X is assume as the perceive value of each passenger. If Type III passengers then they will choose the flight with a lower price if $X \geq (p_A \wedge p_B)$. In case $p_A = p_B$ and $X \geq p_A$, the passenger prefers choose flight A with probability $\beta \in [0, 1]$ and prefers flight B with probability $(1-\beta)$.

Thus, the probability of type I passenger to buy ticket flight A is $\bar{F}(p_A) = 1 - F(p_A)$; the probability of type II passenger to buy ticket flight B is $\bar{F}(p_B) = 1 - F(p_B)$; The probability of type III passenger to buy ticket either flight A or B is $\bar{F}(p_A \wedge p_B)$; The effective arrival rates of flight A and B are then described as follows:

$$\begin{aligned} & (\lambda_1 + \beta \lambda_3 I_{\{p_A=p_B\}} + \lambda_2 I_{\{p_A < p_B\}}) \bar{F}(p_A) \text{ and} \\ & (\lambda_2 + (1-\beta) \lambda_3 I_{\{p_A=p_B\}} + \lambda_3 I_{\{p_A < p_B\}}) \bar{F}(p_B) \end{aligned} \quad (6)$$

Where $I_{\{\text{condition}\}}$ is an indicator function with value 1 if condition is fulfilled, and 0 if otherwise.

This model assumes that the airline faces a Markov decision process. The airline seeks the minimization of the total expected revenue denoted by $R_t(n_1, n_2)$, over remaining horizon $[t, T]$ where n_1 ($0 \leq n_1 \leq C_1$) and n_2 ($0 \leq n_2 \leq C_2$) represent the number of remaining seats of each flight respectively at the time $t \in [0, T]$. The airline then will dynamically adjust the selling prices of the parallel flights in order to maximize revenue.

Supposed that the entire booking horizon $[0, T]$ were divided into small intervals with equal length (Δt) . Each of which will be called a period. It is assumed that there is no more than one passenger arrives over a period $(\Delta t = 1)$. If λ represents the probability of arrival potential passenger in each period, thus, the value is $0 \leq \lambda \leq 1$. At the same time, the potential passenger will buy based on their preference. If the passenger decides to buy a ticket, then the airline will collect the corresponding revenue and consume one unit of seat; otherwise the passenger losses.

The booking horizon $[0, T]$ can be divided into two main general periods: period $t < T$ and period T . Period T is defined the last period before the departure time. Since, the airline can only sell at most one unit of seat; thus, there may be remaining seats with no salvage value in the period T .

At the beginning of each period (in period $t < T$ or in period T), the airline management needs to decide the optimal prices of the flights. The decision will be based on the forecasting of future demand and the current available seats.

Based on the number of available seat (n_1, n_2) , there are four cases considered as follows.

- Case 1 if the tickets in both flights were sold out $(n_1 = n_2 = 0)$.
- Case 2 if flight A has sold out the ticket, and flight B have remaining seats $(n_1 = 0; n_2 > 0)$. The decision variable is flight B's selling price.
- Case 3 if flight B has sold out the ticket while flight A have a number of unsold seats $(n_1 > 0; n_2 = 0)$. The decision variable is flight A's selling price.
- Case 4 if both tickets are still available $(n_1 > 0, n_2 > 0)$. A joint pricing decision is required.

Enhanced Model

In this paper, we will explain our enhancement to the basic model of Xiao *et al.* [11]. Instead of only single full-fare class, we consider multiple full-fare class. Xiao *et al.* [11] only proposed model for an opened single fare class with k different prices, meanwhile we proposed m fare classes. Each of which, we consider k different prices.

Again, supposed that there two flights, A and B, are the parallel flights owned by the same airline company. It is assumed that that both flights have the same seat capacity $(C_a = C_b)$. We also assume that passengers are willing to pay more expensive when they booked the ticket lately. Thus, the airline

set m different full prices and the fare is higher if it is closed to the departure time.

For both flights, the airline set m_a and m_b full-fare classes to open for flight A and B, respectively. Let i_a and i_b be the indexes of opened class for flight A and B respectively $[(i_a \in 1..m_a)$ and $(i_b \in 1..m_b)]$. The full-fare of flight A is represented by FF_{i_a} and FF_{i_b} denotes full-fare of flight B.

Supposed that there are three full-fare classes for both flights $(m_a = m_b = 3)$. Let FF_1 be the highest full-fare class and FF_2 be the lowest one. Thus, for both flights, $FF_1 > FF_2 > FF_3$.

For each full-fare class, we set k alternative prices. k is the number of different prices in each class i_a and i_b . Let $P = \{p_1, p_2, \dots, p_k\}$ be the set of alternative prices where p_1 is the full price and p_k is the lowest possible price. Without loss any generalization, we define that $p_1 > p_2 > \dots > p_k$. If $P_{i_a}^A$ and $P_{i_b}^B$ represent the price of class i opened in Flight A and B then $P_{i_a}^A = (P \times FF_{i_a})$ and $P_{i_b}^B = (P \times FF_{i_b})$.

The probability of type I passenger to buy ticket flight A in class i_a is $\bar{F}(P_{i_a}^A) = 1 - F(P_{i_a}^A)$; the probability of type II passenger to buy ticket flight B in class i_b is $\bar{F}(P_{i_b}^B) = 1 - F(P_{i_b}^B)$. The probability of type III passenger to buy ticket flight A or B in class i_a and i_b is $\bar{F}(P_{i_a}^A \wedge P_{i_b}^B)$.

Thus, the effective arrival rates of flight A and B are as follows

$$\left(\lambda_1 + \beta \lambda_3 I_{\{P_{i_a}^A = P_{i_b}^B\}} + \lambda_2 I_{\{P_{i_a}^A < P_{i_b}^B\}} \right) \bar{F}(P_{i_a}^A) \text{ and} \\ \left(\lambda_2 + (1 - \beta) \lambda_3 I_{\{P_{i_a}^A = P_{i_b}^B\}} + \lambda_3 I_{\{P_{i_a}^A < P_{i_b}^B\}} \right) \bar{F}(P_{i_b}^B) \quad (7)$$

Where $I_{\{\text{condition}\}}$ is an indicator function with value 1 if condition holds, and 0 if otherwise. The joint pricing model at period T if $(n_1 > 0, n_2 > 0)$ is:

$$R_T(n_1, n_2) = \max_{P_{i_a}^A, P_{i_b}^B} \left\{ \left(\lambda_1 + \beta \lambda_3 I_{\{P_{i_a}^A = P_{i_b}^B\}} + \lambda_2 I_{\{P_{i_a}^A < P_{i_b}^B\}} \right) \bar{F}(P_{i_a}^A) + \left(\lambda_2 + (1 - \beta) \lambda_3 I_{\{P_{i_a}^A = P_{i_b}^B\}} + \lambda_3 I_{\{P_{i_a}^A < P_{i_b}^B\}} \right) \bar{F}(P_{i_b}^B) P_{i_b}^B \right\} \quad (8)$$

The joint pricing model at the period $t < T$, if $(n_1 > 0, n_2 > 0)$ is

$$R_t(n_1, n_2) = \max_{P_{i_a}^A, P_{i_b}^B} \left\{ Y(P_{i_a}^A, P_{i_b}^B) \right\} + R_{t+1}(n_1, n_2) \quad (9)$$

where

$$Y(P_{i_a}^A, P_{i_b}^B) = \left(\lambda_1 + \beta \lambda_3 I_{\{P_{i_a}^A = P_{i_b}^B\}} + \lambda_2 I_{\{P_{i_a}^A < P_{i_b}^B\}} \right) \bar{F}(P_{i_a}^A) \times \left[P_{i_a}^A + R_{t+1}(n_1 - 1, n_2) - R_{t+1}(n_1, n_2) \right] + \left(\lambda_2 + (1 - \beta) \lambda_3 I_{\{P_{i_a}^A = P_{i_b}^B\}} + \lambda_3 I_{\{P_{i_a}^A < P_{i_b}^B\}} \right) \bar{F}(P_{i_b}^B) \times \left[P_{i_b}^B + R_{t+1}(n_1, n_2 - 1) - R_{t+1}(n_1, n_2) \right] \quad (10)$$

Different from the single fare class model of Xiao *et al.* [11] where the seat allocation is a given parameter, in multiple fare classes, we need to previously determine the seat allocation for each fare class. To solve this issue, we previously used the nested EMSR approach model of Belobaba [2]. The output of this model is the seat allocation for each class of Flight A and B. Similar to Xiao *et al.* [11], we also used backward dynamic program-ming to solve the problems.

Result and Discussion

Seat Allocation Control

We used the following parameter in the numerical experiments: There are 30 seats available f discounted prices ($k=6$). So the feasible set price is or each flight ($C_a = C_b = 30$ seats), and both flights opened 3 full-fare classes. For each of which, the ticket offered in a full price and five $P = (1, 0.9, 0.8, 0.7, 0.6, 0.5)$. The probabilities that a potential passenger's perceive value of above set selling price are (0.3, 0.4, 0.48, 0.58, 0.7 and 0.85) respectively. We have divided the entire horizon into $T = 100$, and the arrival rate of potential passenger is $\lambda = 0.75$.

We will discuss the model in three conditions.

Condition 1:

In condition 1, we assume that the price of each classes is same for both flights; ($FF_{ia} = FF_{ib}$ for all i), and also under the same demand distribution. This means that Flight A and B have equal numbers of seat allocation in each fare class. It was sound unrealistic but we can use this condition in order to compare to other conditions. The details of parameter in condition 1 are shown in Table 1.

Based on the nested EMSR approach, we determine seat allocation for each fare class. The seat allocation of each fare class is shown in Table 1. Figure 2 shows the visualization of EMSR for condition 1.

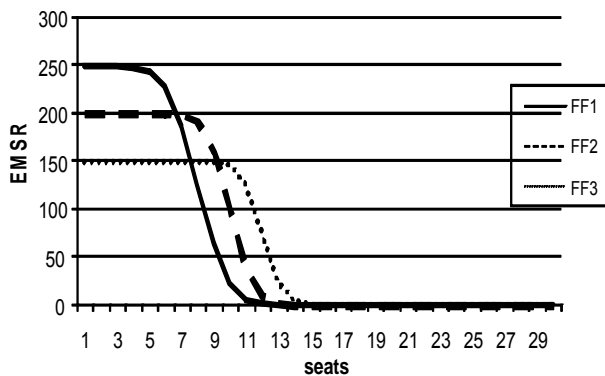


Figure 2. EMSR for flights A and B in condition 1

Table 1. Parameter and seat allocation of the both flights for condition 1

		Flight A & B		
Capacity (seat)		30		
Class (FF_i)		FF_1	FF_2	FF_3
Fare (\$)		250	200	150
Demand distribution	Mean	8	10	12
	SD	1.5	1.2	1
Seat allocation		7	10	13

Condition 2:

In condition 2, we assume that the prices of each class are same for both flights. However, they have different demand distribution. This means that Flight A and B have different number of seat allocation in each fare class. The parameter and seat allocation for this condition are shown in Table 2. We may see that seat allocation of flight A is similar to that of in condition 1

Condition 3:

In condition 3, we assume the fares of each class in flight A and B are different. We assume that the price of flight B is more expensive than that of flight A. With different demand distribution, the seat allocation of each class can be seen in Table 3. The EMSR's visualization for flight B is shown in Figure 4.

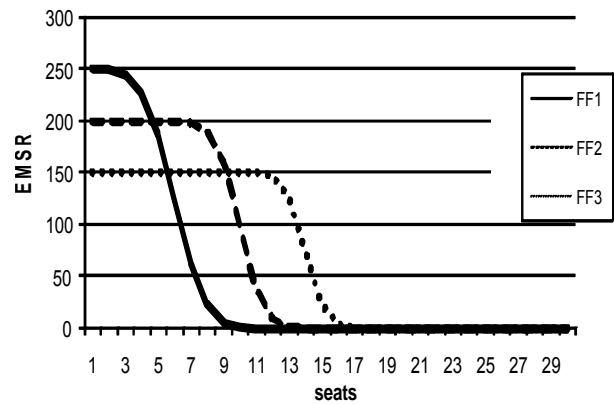


Figure 3. EMSR for Flight B in condition 2

Table 2. Parameter and seat allocation of flight A & B for condition 2

		Flight A		
Class	Fare (\$)	Demand dist		Seat allocation
		Mean	SD	
FF_1	250	8	1.5	7
FF_2	200	10	1.2	10
FF_3	150	12	1.0	13
		Flight B		
Class	Fare (\$)	Demand dist		Seat allocation
		Mean	SD	
FF_1	250	6	1.5	4
FF_2	200	10	1.2	10
FF_3	150	14	1.0	16

Table 3. Parameter and seat allocation of flight A & B for condition 3

Flight A				
Class	Fare (\$)	Demand dist		Seat allocation
		Mean	SD	
<i>FF₁</i>	250	8	1.5	7
<i>FF₂</i>	200	10	1.2	10
<i>FF₃</i>	150	12	1.0	13
Flight B				
Class	Fare (\$)	Demand dist		Seat allocation
		Mean	SD	
<i>FF₁</i>	315	6	1.5	4
<i>FF₂</i>	250	9	1.2	9
<i>FF₃</i>	190	12	1.0	17

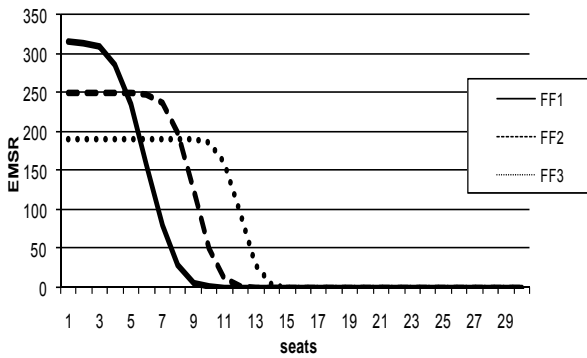


Figure 4. EMSR for Flight B in condition 3

Joint Pricing

After the seat allocation of each fare class has been determined, we then calculated the expected revenue of the joint flights under some different circumstances. First, the numerical experiments using the different arrival rates of Type III passengers (α_3) under the fixed probability of passengers prefer choosing flight A (β).

Different Values of α_3

We computed the optimal total revenue under different values of α_3 from 0 to 1 and keeping the β 's value in 0.5. We set $\alpha_1 = \alpha_2$, so the values of α_1 & α_2 are equal to $\frac{(1-\alpha_3)}{2}$. The expected revenue of flights is shown in Table 4, 5, and 6; respectively for Conditions 1, 2, and 3. We have compared the expected revenues of them. They are represented in Figure 5.

The results showed that proportion of type III passengers have influenced the total expected revenue of both flights. The higher value of α_3 the higher total expected revenue of joint flights. This implies that airline management needs to notice when inflexible time passengers become time flexible passengers, they need to manage the price of both flights in order to control their demand.

Table 4. The expected revenue of flights under different value of α_3 in condition 1

Condition 1				
α_3	$\alpha_1 = \alpha_2$	<i>R_a</i>	<i>R_b</i>	<i>R_{total}</i>
0.0	0.50	2973.9165	2973.9165	5947.8330
0.1	0.45	2996.3259	2996.3259	5992.6519
0.2	0.40	3011.6638	3011.6638	6023.3276
0.3	0.35	3021.7126	3021.7126	6043.4253
0.4	0.30	3028.3787	3028.3809	6056.7595
0.5	0.25	3033.0530	3032.9888	6066.0417
0.6	0.20	3036.4031	3036.4031	6072.8062
0.7	0.15	3039.1101	3038.8000	6077.9102
0.8	0.10	3040.9736	3040.9563	6081.9299
0.9	0.05	3042.6343	3042.6558	6085.2900

Table 5. The expected revenue of flights under different value of α_3 in condition 2

Condition 2				
α_3	$\alpha_1 = \alpha_2$	<i>R_a</i>	<i>R_b</i>	<i>R_{total}</i>
0.0	0.50	2973.917	2806.605	5780.522
0.1	0.45	2987.921	2835.062	5822.983
0.2	0.40	3002.359	2850.854	5853.214
0.3	0.35	3015.547	2858.305	5873.853
0.4	0.30	3029.331	2858.818	5888.149
0.5	0.25	3042.320	2856.173	5898.493
0.6	0.20	3055.663	2850.617	5906.279
0.7	0.15	3068.612	2843.718	5912.330
0.8	0.10	3081.812	2835.434	5917.245
0.9	0.05	3098.275	2823.265	5921.540

Table 6. The expected revenue of flights under different value of α_3 in condition 3

Condition 3				
α_3	$\alpha_1 = \alpha_2$	<i>R_a</i>	<i>R_b</i>	<i>R_{total}</i>
0.0	0.50	2973.917	3500.278	6474.194
0.1	0.45	3050.730	3457.117	6507.846
0.2	0.40	3047.723	3493.441	6541.164
0.3	0.35	3020.185	3549.774	6569.959
0.4	0.30	2985.979	3607.504	6593.483
0.5	0.25	2953.355	3658.108	6611.463
0.6	0.20	2915.051	3709.565	6624.616
0.7	0.15	2889.885	3745.612	6635.497
0.8	0.10	2868.631	3776.945	6645.576
0.9	0.05	2840.368	3815.704	6656.072

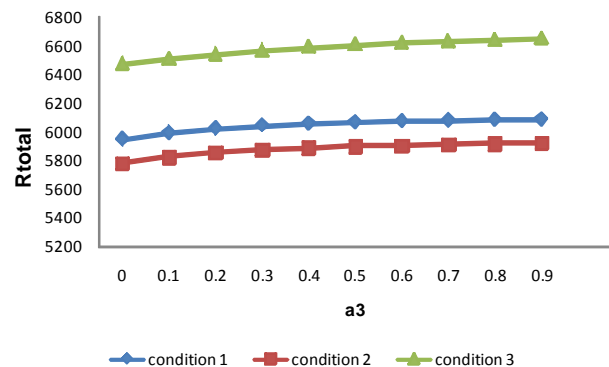


Figure 5. Total revenue in different value of α_3

Specifically, by comparing the results of conditions 1 and 2, we may see that the total expected revenue of condition 1 is higher than that of condition 2. This is due to different demand distribution in each class of flights A and B.

We may make a conclusion that the seat allocation distribution of joint flights has significantly contributed to the total revenue received. Thus, in managerial perspective, the airline management should optimize the seat allocation management of the joint flights. Our proposed model can be used for this purpose.

Different Values of β and Small α_3

We set β values from 0 to 1, whereas $\alpha_3 = 0.1$, and $\alpha_1 = \alpha_2 = 0.45$ are constant. We choose small proportion for leisure passengers (α_3) to represent a condition when time sensitive passengers are more dominant than price sensitive passengers. When selling prices of both flights are identical, the larger value of β implies that type III passengers prefer choose flight A. As a result, for all conditions, the expected revenue of flight A increases.

The total expected revenue under different values of β in condition 1 shown in Table 7 and Figure 6 has formed a curve shape. The highest revenue occurred when $\beta = 0.5$, which is situation where both flights have identical price as well as the seat allocation.

Table 7. R_{total} under different value of β in condition 1

β	Condition 1		
	R_a	R_b	Rtotal
0.0	2908.828	3082.3762	5991.2041
0.1	2925.605	3066.0991	5991.7043
0.2	2942.968	3049.1401	5992.1082
0.3	2960.701	3031.7056	5992.4063
0.4	2978.545	3014.0449	5992.5903
0.5	2996.326	2996.3259	5992.6519
0.6	3014.049	2978.5417	5992.5903
0.7	3031.706	2960.7007	5992.4063
0.8	3049.140	2942.9680	5992.1082
0.9	3066.101	2925.6030	5991.7041
1.0	3082.375	2908.8291	5991.2039

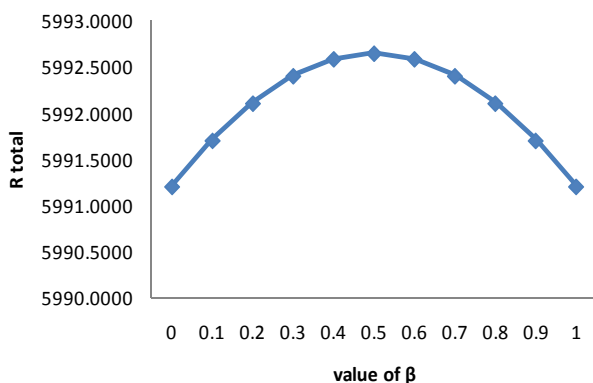


Figure 6. Total revenue in different values of β in condition 1

In condition 2, which is a situation when flights A and B have the same price but in different demand, the results showed that the higher β the higher total expected revenue.

Different situation was occurred in condition 3. In this condition, we have set that the price of flight B is higher than that of flight A. The result is that the higher β the lower total expected revenue.

These two experiments results imply that the airline will gain higher revenue if they set price of flight A higher than flight B, when the time flexible passenger prefers to choose flight A. Tables 8 and 9 represent total expected revenue of condition 2 and 3 under different values of β . Figures 7 and 8 represent these two conditions in graphics respectively.

Table 8. R_{total} under different value of β in condition 2

β	Condition 2		
	R_a	R_b	Rtotal
0.0	2912.733	2903.667	5816.400
0.1	2926.528	2891.314	5817.843
0.2	2940.864	2878.382	5819.245
0.3	2956.030	2864.555	5820.584
0.4	2971.646	2850.190	5821.836
0.5	2987.921	2835.062	5822.983
0.6	3004.570	2819.433	5824.003
0.7	3021.248	2803.638	5824.886
0.8	3037.945	2787.682	5825.627
0.9	3054.552	2771.668	5826.220
1.0	3071.302	2755.379	5826.682

Table 9. R_{total} under different value of β in condition 3

β	Condition 3		
	R_a	R_b	Rtotal
0.0	3048.474	3459.577	6508.051
0.1	3049.391	3458.575	6507.966
0.2	3049.742	3458.170	6507.912
0.3	3050.056	3457.823	6507.879
0.4	3050.388	3457.474	6507.861
0.5	3050.730	3457.117	6507.846
0.6	3050.682	3457.155	6507.836
0.7	3050.897	3456.935	6507.831
0.8	3051.233	3456.595	6507.828
0.9	3051.331	3456.496	6507.827
1.0	3051.287	3456.540	6507.827

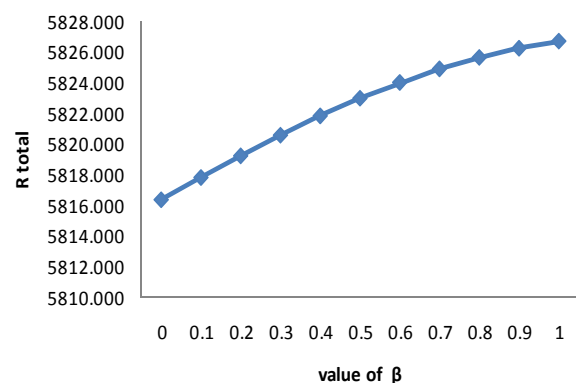


Figure 7. Total revenue in different values of β in condition 2

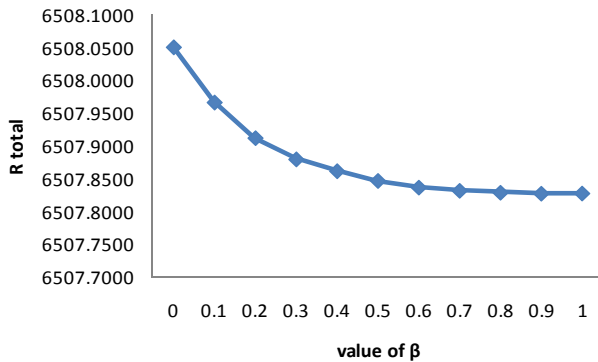


Figure 8. Total revenue in different values of β in condition 3

Conclusion

In this paper, we have extended the model of Xiao *et al.* [11] of Joint Pricing Model for Parallel Flights. Instead of single full-fare class, we have considered the cases of multiple full-fare class. In this enhancement, we have employed the model of nested Expected Marginal Seat Revenue (EMSR) approach to previously define the seat allocation for each fare class. We have also modified the dynamic programming algorithm used by Xiao *et al.* [11]. Some numerical experiments have been conducted. Our experiment results has showed that the total expected revenue of both flights induced by proportion of time-flexible passengers (type III passengers) and the number of allocated seat in each full-fare class.

When the number of time-flexible passengers tends to increase, the airline has to set the price, in order to control the allocation of demand between two flights. The airline can gain higher revenue by set the price of full-fare class of one flight higher than another.

In general, we may conclude that our proposed model has shown that the closed relationships between demand management, which is represented by the price of each fare class, and total expected revenue considering the passenger choice behavior.

In this paper, we have not yet considered overbooking, cancellation and no show problems happened in reality condition. The future research of parallel flights problem can be considered in those situation.

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