

FUZZY MODELING APPROACH AND GLOBAL OPTIMIZATION FOR DUAL RESPONSE SURFACE

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ABSTRACT

Dual Response Surface (DRS) with Lagrange multiplier is one of the most familiar classical multi response surface methods. Classical DRS optimization doesn't concern about the quality characteristic of responses. In this paper, fuzzy approach is proposed for modeling DRS and quality characteristic of response simultaneously. The proposed method represented the object's quality characteristic physically. The proposed method is applied to composite carbon drilling process and resulting nonlinear function that to be determined its optimal point. Many optimization methods fail to reach global optimum point because the non linear function is multimodal. Therefore, we used genetic algorithm for finding the global optimum point.

Keywords: dual responses surface, quality characteristics, fuzzy, global optimum, genetic algorithms.

1. INTRODUCTION

Response Surface Methodology (RSM) is able to find the optimal setting for input variables that maximizes (or minimizes) the responses (Khuri and Cornell, 1996). RSM consists of roughly three stages, they are, data collection (design of experiment), model building and optimization. A complete and detail explanation about design experiment for the first and second order model are referred to Khuri and Cornell (1996) and Myers and Montgomery (2002).

Dual Response Surfaces requires an overall optimization because interrelationships may exist among the responses and separate optimization will give meaningless result (Khuri and Cornell, 1996). DRS was firstly introduced by Myers and Carter in 1973. They used Lagrange multiplier approach. This classical DRS doesn't concern about the quality characteristic of responses. Some researchers have modified DRS to accommodate quality characteristic of response in the optimization. Del Castillo and Montgomery (1993) optimized DRS using advanced computational algorithm to avoid dimensionality problem. Lin and Tu (1995) pointed out that the optimization scheme based on Lagrange multiplier can be misleading due to the unrealistic restriction of forcing the estimated prime response to specific value. Consequently, they proposed a new objective function to be minimized, square of estimated secondary response was added by square of distance of the estimated prime response to its target (similar with loss function). Copeland and Nelson (1996) optimized DRS via direct function minimization.

We proposed a novel mathematical programming formulation for DRS based on fuzzy optimization methodology. This method is applied to composite carbon drilling process (Isnaini,

2000). There are two responses (drill wear and hole's roundness deviation) and three control factors (x_1, x_2, x_3). Drill wear is primary response and hole's roundness deviation is secondary response. Both responses have quality characteristic Smaller the Better (STB), it means that the values of the response are expected to be as small as possible.

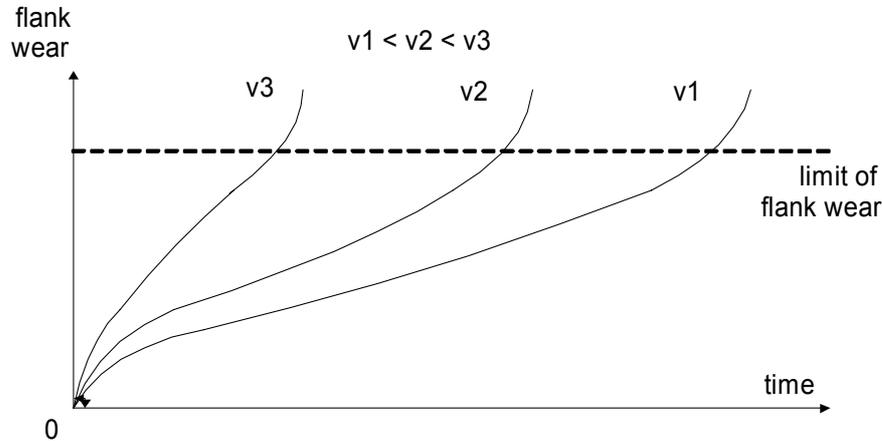


Figure 1. The Growth of Flank Wear at Some Speeds of Cutting (v_i)

In fact, the growth of the drill wear is not linear (Figure 1). The growth of drill wear is fast immediately after the drill is used. Several times after the drill is used, the growth of the drill wear is linear. After the limit of the flank wear, the growth of the drill wear is faster than before. The growth of the drill wear after the limit of the flank wear is the same as the growth of the drill wear immediately after it is used (Rochim, 1993). Different speeds of cutting result in the same behavior of growth with different time consuming to reach limit of the flank wear.

Classical RSM gives the linear change of the degree of satisfaction. If it is not practically valid (nonlinear or even non symmetric) classical RSM can be misleading (Kim and Lin, 1998). In this work, we propose a fuzzy approach to tackle this problem. The primary response, drill wear, affected the secondary response, hole's roundness deviation. We assume secondary response's behavior is same as the primary response. Based on this real condition, classical RSM can not tackle the drilling process problem. The membership function (MF, as in fuzzy set theory) of a response is interpreted as the degree to which a response satisfies its target. MF is between 0 and 1 (0 being the worst and 1 being the best).

2. DUAL RESPONSE SURFACE

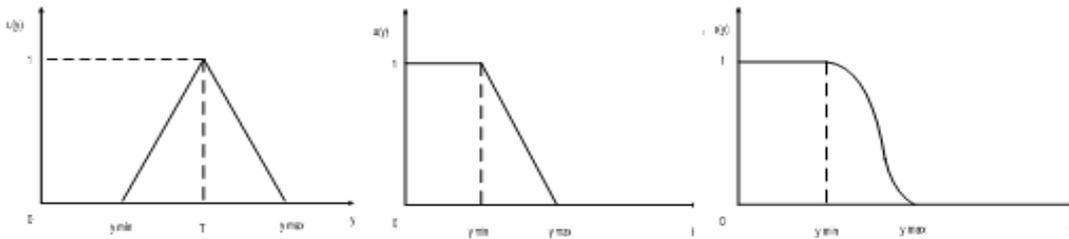
An experiment in which a number of responses are measured simultaneously for each setting of a group of input variables is called a multi response experiment. The detail explanation of a linear multi response model can be seen in Khuri and Cornell (1996) and Myers and Montgomery (2002). The Best Linear Unbiased Estimator (BLUE) of linear multi response model coefficient is the same as the Ordinary Least Square (OLS) estimator which is obtained from fitting the i^{th} model individually (Khuri and Cornell, 1996). Myers and Carter considered an optimization problem associated with two responses; one called primary response, and the other is called secondary response. In this method, secondary response function imposes certain constraints on the optimization of the primary response function.

3. FUZZY SET AND MEMBERSHIP FUNCTION ASSESSMENT

If X is a collection of objects denoted generically by x , then a fuzzy set A is defined as a set of ordered pairs $A = \{(x, \mu_A(x)) \mid x \in X\}$, where $\mu_A(x)$ is called membership function (or MF for short) for the fuzzy set A . The MF maps each elements of X to a membership grade (or membership value) between 0 and 1. For simplicity of notation, we introduced an alternative way of denoting a fuzzy set. A fuzzy set A can be denoted as follows:

$$A = \begin{cases} \sum_{x_i \in X} \frac{\mu_A(x_i)}{x_i}, & \text{if } X \text{ is a collection of discrete objects} \\ \int_X \frac{\mu_A(x)}{x} & \text{, if } X \text{ is a continuous space (usually real)} \end{cases} \quad (1)$$

There are several MF such as: π , bell, Gaussian, trapezoidal, S and T (triangular), etc. A detail explanation of the shape and formulae about MF are referred to Jang, Sun, and Mizutani (1997) and Zimmermann (2000).



(a) & (b) Linear MF of Response ; (c) Non Linear MF of Response

Figure 2. Membership Function of Response

If the degree of satisfaction doesn't change linearly, in other word a non linear MF is desired, then the process of selecting an admissible functional form is difficult and time consuming. However, it can be simplified by employing a general functional form which can generate a rich variety of shape by adjustment of its parameters. Kim and Lin (1998) built a general non linear MF that is called an exponential function. This function has been proven to provide a reasonable and flexible representation of human perception (Kirkwood and Sarin (1980) in Kim and Lin (1998)). The exponential function is defined as:

$$\mu(z) = \begin{cases} \frac{e^d - e^{d|z|}}{e^d - 1}, & \text{if } d \neq 0 \\ 1 - |z|, & \text{if } d = 0 \end{cases} \quad (2)$$

where d is a constant ($-\infty < d < \infty$), z is a standardized parameter representing the distance of the response from its target in units of the maximum allowable deviation. For a symmetric case, z can be defined as Nominal the Best (NTB), Smaller the Better (STB), and Larger the Better (LTB). For a NTB case, response is expected equals to the target (T_i) as it can be seen in Figure 2(a). The MF can be expressed as

$$z_i = \frac{\hat{y}_i - T_i}{y_i^{\max} - T_i} = \frac{\hat{y}_i - T_i}{T_i - y_i^{\min}} \quad \text{for } y_i^{\min} < \hat{y}_i \leq y_i^{\max} \quad (3)$$

A STB case, response is expected as small as possible (y^{\min}) as it can be seen in Figure 2(b) which is stated as

$$z_i = \frac{\hat{y}_i - y_i^{\min}}{y_i^{\max} - y_i^{\min}} \quad \text{for } y_i^{\min} < \hat{y}_i \leq y_i^{\max} \quad (4)$$

A LTB case is the reverse of STB. For an asymmetric case, a NTB case as in equation (3) can be easily modified to be an asymmetric function, i.e. function $f(y_i^{\max} - T_i)$ and $f(T_i - y_i^{\min})$ had different shape.

The union, intersection, and complement are the most basic operations on classical sets. Corresponding to the classical set operation, fuzzy sets have similar operations (Jang, Sun, Mizutani, 1997) and (Zimmermann, 2000). Sometimes, it is advantageous or necessary to use MF with two inputs, each in a different universe of discourse. MFs of this kind are generally referred to as two-dimensional MF. Ordinary MF (MF with one input) is referred to as one dimensional MF. One natural way to extend one-dimensional MF to two-dimensional ones is via cylindrical extension (Jang, Sun, Mizutani, 1997).

4. GENETIC ALGORITHM

Genetics Algorithms (GA) is one of the derivative-free stochastic optimization methods based loosely on the concepts of natural selection and evolutionary process (Jang, Sun, and Mizutani, 1997). GA encodes each point in parameter (or solution) space into a binary bit strings called a *chromosome*. Each point is associated with a *fitness* value that, for maximization, is usually equal to the objective function evaluated at the point. Instead of single point, GA usually keep a set of points as a *population* (or *gene pool*), which is then evolved repeatedly toward a better overall fitness value. In each *generation*, the GA constructs a new population using *genetic operators* such as *crossover* and *mutation*.

GA differs from conventional optimization and search procedures in several fundamental ways. These are the summaries (Gen and Cheng, 1997):

1. GA work with a coding of solution set, not the solutions themselves.
2. GA search from a population of solution, not a single solution.
3. GA use payoff information (fitness function), not derivatives or other auxiliary knowledge.
4. GA use probabilistic transition rules, not deterministic rules.

5. MODELING AND ANALYSIS

The proposed method was applied to drilling process of composite carbon type 3K-70-PW from IPTN Bandung, Indonesia (Isnaini, 2000), which was also used in Akbar and Setiawan (2005). The purpose of the experiment was to determine the effect of speed of machine's rotation (X_1), feeding (X_2), and angle (X_3) to the both responses. The experiment was conducted in Central Composite Design (CCD).

5.1 Fitting Second Order Model

The plots of primary response (y_1) spread symmetrically and there is no indication of outlier. However, the plots of secondary response (y_2) don't spread symmetrically. There is an extreme point that indicates the presence of an outlier ($y_2 = 33$). We tested the outlier using DFFITS, DFBETAS, and Cook's distance (Rousseeuw and Leroy, 1987). They showed that $y_2=33$ is an outlier. The second order model for both y_1 and y_2 were fitted as follows:

$$\hat{y}_1 = 0.23624 + 0.030274 z_1 + 0.027951 z_2 + 0.22875 z_3 + 0.073398 z_1^2 + 0.073229 z_2^2 + 0.052822 z_3^2 - 0.00525 z_1 z_2 + 0.02562 z_1 z_3 + 0.03625 z_2 z_3 \quad (5)$$

Model in equation (5) was estimated by using OLS in coded value. We substituted these variables below into equation (5), so we got equation (6).

$$z_1' = \frac{(x_1 - 700)}{250} \quad z_2' = \frac{(x_2 - 60)}{30} \quad z_3' = \frac{(x_3 - 75)}{5}$$

$$\hat{y}_1 = 14.63989212 - 0.0030182192 x_1 - 0.0264672 x_2 - 0.3412042 x_3 + 0.000001174368 x_1^2 + 0.0000813655556 x_2^2 + 0.00211288 x_3^2 - 0.0000007 x_1 x_2 + 0.000020496 x_1 x_3 + 0.00024167 x_2 x_3 \quad (6)$$

Hole's roundness deviation (y_2), which is influenced by an outlier, was fitted by using Least Median Square (LMS) regression (7).

$$\hat{y}_2 = 341.91081 - 0.143124 x_1 + 0.0103172 x_2 - 7.792715 x_3 + 0.0000454 x_1^2 - 0.0002 x_2^2 + 0.048 x_3^2 - 0.000127 x_1 x_2 + 0.0011636 x_1 x_3 + 0.00097 x_2 x_3 \quad (7)$$

Now, we got new models ((6) and (7)) that will be used for the next analysis.

5.2 Response Quality Characteristic and Its Specification

Primary and secondary responses are expected to have a value as small as possible (STB). In practical, both responses have upper and lower bounds (specification). These bounds are useful to determine the membership function's range of fuzzy set. Rochim (1993) stated that the upper bound of drill type HSS is 0.3–0.8 mm. In this paper we used 0.3 mm and 0 mm as our upper bound and lower bound respectively. The data were collected from drilling process using drill with diameter 12 mm. The diameter of a drill determines the upper and lower bound of hole's roundness deviation (Sato and Sugiarto, 1999). The upper and lower bounds of y_2 are 18 micron and 0 micron.

5.3 Membership Function Assessment

As discussed in the previous section, the proposed approach requires that the membership function of the first and second responses be specified. Linear membership functions, as in (3) and (4), were used when the marginal rate of change of membership values, $\mu(z_i)$, was constant. Marginal rate of change of membership values mean the change of the degree of satisfaction for each response when its value changed. If the marginal rate of change of membership values was not constant, a nonlinear MF should be employed.

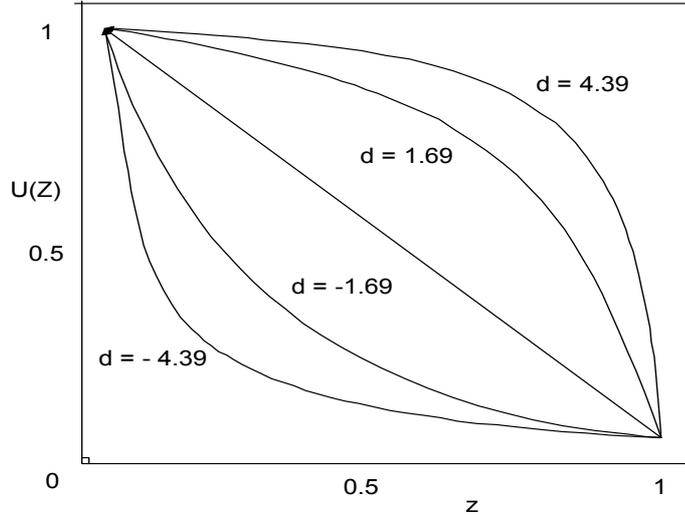


Figure 3. Membership Function Exponential

In fact, the growth of drill wear is not constant, so we employed the nonlinear MF. The exponential function is suitable to explain the characteristic of drill wear's growth. The z value in the exponential function interprets the standardized distance of response to its target. In this work, we used the target value as small as possible (zero). For the target to be equal to zero, we got the following values:

$$z_1 = \frac{\hat{y}_1}{0,3} \tag{8}$$

$$z_2 = \frac{\hat{y}_2}{18} \tag{9}$$

The MF of the responses were given to be

$$\mu(z_1) = \begin{cases} \frac{e^d - e^{\frac{d|\hat{y}_1|}{0,3}}}{e^d - 1}, & \text{if } d \neq 0 \\ 1 - \frac{|\hat{y}_1|}{0,3}, & \text{if } d = 0 \end{cases} \tag{10}$$

$$\mu(z_2) = \begin{cases} \frac{e^d - e^{\frac{d|\hat{y}_2|}{18}}}{e^d - 1}, & \text{if } d \neq 0 \\ 1 - \frac{|\hat{y}_2|}{18}, & \text{if } d = 0 \end{cases} \tag{11}$$

At an arbitrary point z_0 , ($0 < z_0 < 1$), the decision maker assesses the degree of satisfaction, denoted as s , and then solves the equation $\mu(z_0) = s$ for d . There is no closed form solution to (2); hence it must be solved numerically. However, when $z_0 = 0,5$, the d is obtained as

$$d = 2 \ln \left(\frac{s}{1-s} \right) \tag{12}$$

The expected degree of satisfaction for the y_1 was 0.9, so we got $d = 4.39$ for y_1 . Expected degree of satisfaction of y_2 was 0.7, by the same way we got $d = 1.69$. The expected degree of satisfaction for each response is based on the degree of importance of response.

A dual response problem requires an overall optimization, that is, a simultaneous satisfaction with respect to both primary and secondary responses. A ‘minimum’ operator of fuzzy set is employed for aggregating the two objectives, where $\mu(\hat{y}_1)$ and $\mu(\hat{y}_2)$ are function of x (i.e. fitted response surface) and Ω defines the feasible region of x . The above formulation aims to identify x which would maximize the minimum degree of satisfaction, with respect to both responses within the feasible region, that is, maximize $[\min(\mu(\hat{y}_1), \mu(\hat{y}_2))]$ with respect to $x \in \Omega$. The complete formulation of drilling process problem is formulated as follow:

$$\text{Objective function} = \max \{ \min(\mu(z_1), \mu(z_2)) \} \tag{13}$$

$$\text{Subject to} = \frac{e^{4.39} - e^{4.39|\frac{\hat{y}_1}{0.3}|}}{e^{4.39} - 1} \geq \max \{ \min(\mu(z_1), \mu(z_2)) \} \tag{14}$$

$$\frac{e^{1.69} - e^{1.69|\frac{\hat{y}_2}{18}|}}{e^{1.69} - 1} \geq \max \{ \min(\mu(z_1), \mu(z_2)) \}$$

$$0 \leq z_1 \leq 1 \quad \text{and} \quad 0 \leq z_2 \leq 1$$

$$450 \leq x_1 \leq 950$$

$$30 \leq x_2 \leq 90$$

$$70 \leq x_3 \leq 80$$

5.4 Optimization

Formulation (13) and (14) represent a constrained nonlinear optimization with a single objective. In principle, any general algorithm for nonlinear problem can be used, including General Reduced Gradient (GRG) method (Del Castillo and Montgomery (1993) and Rardin (1998)). It should be noted that GRG algorithm may fail to reach an optimal point if the membership function in constraint function (14) has non-differentiable points. Moreover, the nonlinear optimization may result in local optimum because the found solution can depend on the starting point supplied by the user.

In this work we used the simplest GA to find global optimal point. First, we encoded the decision factors (X_1 , X_2 , and X_3) into a *binary string*. The length of the string depends on the required precision. We use three decimal points precision. The required bits for a variable with the determined precision can be seen in (Gen and Chen, 1997). Number of bits which are required for X_1 was calculated as follow:

$$(950 - 450) \times 1000 = 500.000$$

$$2^{18} < 500.000 < 2^{19}, m_1 = 19$$

Factor X_1 requires 19 bits. By the same way we get the length of required bits for X_2 and X_3 , they are 16 bits and 14 bits. We got the total length of a binary string, called *chromosome*, was 49 (=19+16+14) bits. A chromosome represents a solution point. A set of solution point is called *population*.

The initial population (75 chromosomes) was selected based on *roulette wheel* approach (Gen and Chen, 1997). We used one-cut-point *crossover* method with mutation rate 0.01. If mutation rate is high, above 0.1, GA performance will approach that of a primitive random search (Jang, Sun, and Mizutani, 1997). Now we have just completed an iteration of GA, called one *generation*. The test runs were terminated after 100, 250, and 500 generations. We changed the probability of crossover from 0.25, 0.50, and 0.75.

From those experiments, we got the optimum condition on $y_1=0.228$ mm and $y_2=9.8$ micron. This condition was resulted from $X_1=624.284$ rpm, $X_2=52.885$ mm/min, and $X_3=74.144^0$ (Table 1.). The result of fuzzy approach was the smallest drill's wear with degree of overall satisfaction 65.8% (Table 2). This value is better than classical DRS (Isnaini, 2000) which having degree of overall satisfaction 64.9% and Lin & Tu approach (Akbar and Setiawan, 2005) with degree of satisfaction 27.1%.

Table 1. Optimal Points from Several Optimization Methods

Methods	Optimal				
	Predictors			Responses	
	X_1	X_2	X_3	Y_1	Y_2
Lagrange multiplier (Isnaini, 2000)	619,7	59,3	73,7	0,230	9,674
Lin & Tu (Akbar and Setiawan, 2005)	798,5	90	70,6	0,279	7,599
Fuzzy approach	624,284	52,885	74,144	0,228	9,800

Table 2. Standardized Deviation, Individual and Overall Satisfaction of Responses.

Methods	Standardized deviations		Degree of Satisfaction of responses		Overall satisfaction
	Y_1	Y_2	Y_1	Y_2	
	Lagrange multiplier	0,767	0,537	0,640	
Lin & Tu	0,929	0,422	0,271	0,764	0,271
Fuzzy approach	0,761	0,544	0,658	0,658	0,658

On individual optimization of y_2 , fuzzy modeling approach results the larger response which contradict to characteristics quality (STB). However, this value falls in the specification area. So we focused on simultaneous optimization which optimizes two responses simultaneously. Because of it, we should use overall satisfaction as measurement of goodness. In this study, fuzzy modeling approach gave the highest degree of satisfaction. Moreover, fuzzy modeling approach could accommodate the quality characteristic of responses. This aspect is more reasonable to be applied.

6. CONCLUSION

A fuzzy modeling approach to optimize DRS has been presented. The proposed approach aims to identify a set of process parameter condition to simultaneously maximize the degree of

satisfaction with respect to both responses. A fuzzy modeling approach concern with some criteria. First, it concerns about the quality characteristic of responses. The second, fuzzy approach concerns about the upper and lower bounds (specification) based on practical problem. Another possible advantage of fuzzy approach is that it can optimize the robust regression model. Moreover, a nonlinear membership function can model the object's quality characteristic very flexibly.

7. FUTURE WORKS

More suitable membership function is the concern of the future work. We should find the detail condition of the control factors when the drill wear behaves as in Figure 1. The MF should be built based on this detail condition. A constrained optimization problem in GA should be studied more intensive, especially in handling constraints.

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